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Applied Mathematics 20

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Module

7

Applied Mathematics 20

MEASUREMENT

Learning
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Alberta
LEARNING

Applied Mathematics 20
Module 7: Measurement
Student Module Booklet
Learning Technologies Branch
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Teachers	✓
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Other	

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- Alberta Learning, **http://www.learning.gov.ab.ca**
- Learning Technologies Branch, **http://www.learning.gov.ab.ca/ltb**
- Learning Resources Centre, **http://www.lrc.learning.gov.ab.ca**

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Welcome

Applied Mathematics 20

Module 1
Graphs

Module 2
Non-Linear Functions

Module 3
Linear Systems

Module 4
Linear Programming

Module 5
Finance

Module 6
Circles

Module 7
Measurement

Welcome to Module 7. We hope you'll enjoy your study of Measurement.

Applied Mathematics 20 contains seven modules and a final test. Work through the modules in the order given, since several concepts build on each other as you progress through the course.

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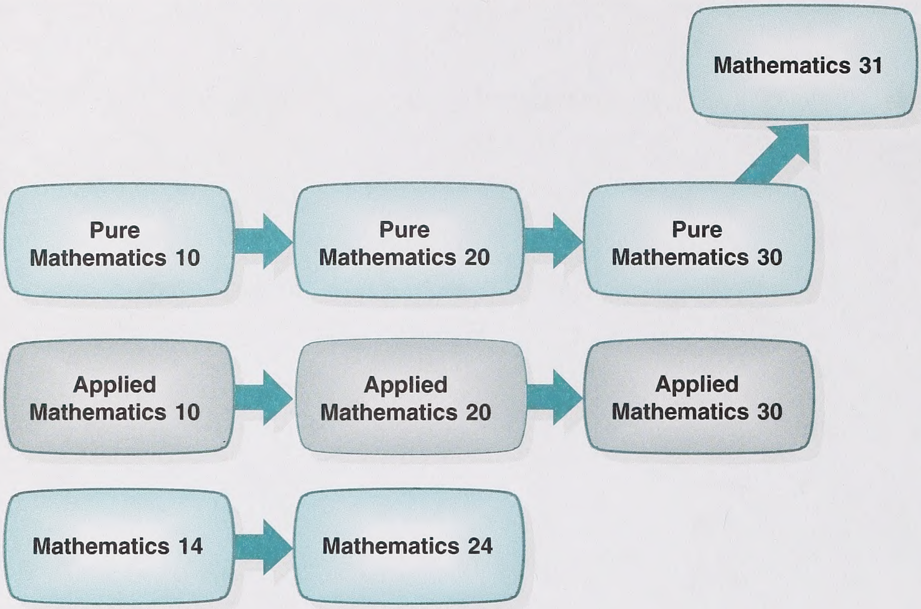
Introduction to Applied Mathematics 20

Applied Mathematics 20 is the second course in the Applied Mathematics 10–20–30 program of studies. Another program of studies is Pure Mathematics 10–20–30; students who complete Pure Mathematics 30 often choose to take Mathematics 31. A third program of studies is Mathematics 14–24.



Each mathematics program is designed for students with different mathematical strengths and interests.

- Pure Mathematics 10–20–30 is intended for students who are strong in algebra and mathematical theory.
- Applied Mathematics 10–20–30 is better suited to students who prefer to solve problems using numerical reasoning or geometry.
- Mathematics 14–24 is a general mathematics program for high school students who have experienced difficulties in previous mathematics courses.

Each sequence of courses is designed for students with different career plans. For example, Pure Mathematics 30 is a prerequisite for admission to many university programs. Many colleges and technical institutes, however, will admit students who have successfully completed Applied Mathematics 30.

You may find it helpful to read “Questions and Answers About Senior High School Mathematics” and “The New Senior High School Mathematics Program and Post-Secondary Studies.” These documents can be found at the following Internet site:

http://www.learning.gov.ab.ca/k_12/curriculum/bySubject/math

Before enrolling in Applied Mathematics 20, it is recommended that you talk with a school counsellor about your career plans.

TRANSFERRING FROM THE APPLIED PROGRAM

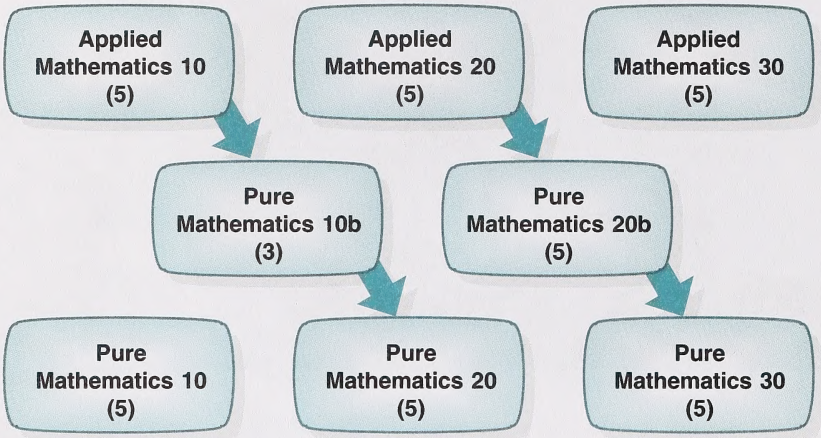
You should be aware that the applied and pure mathematics courses do have some topics in common; other topics are independent.

The following table shows some common and independent topics.

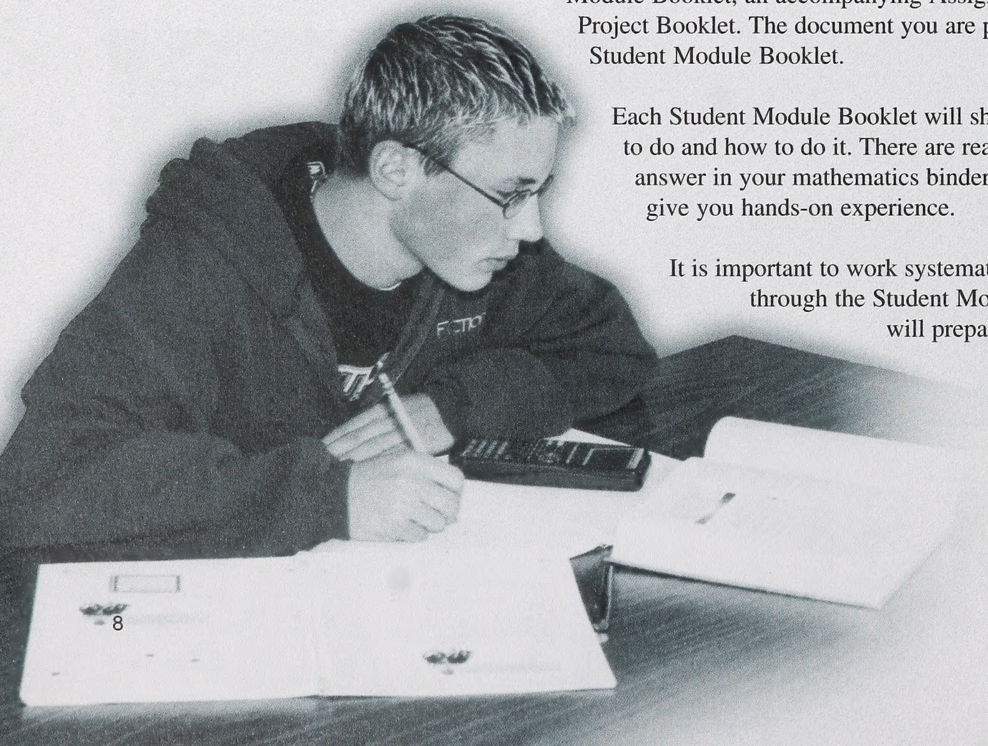
Applied Topics	Common Topics	Pure Topics
• linear programming • data tables and trends • design and layout • metric and imperial measure • data presentation • vectors and matrices • periodic, fractal, and recursive patterns • financial decision making • costing and design problems	• spreadsheets • line segments and linear graphs • scaling • triangles • surveys • financial mathematics • quadratic functions • circle geometry • the bell curve	• irrational numbers • exponents • polynomial and rational expressions • mathematical expectation • growth patterns • linear and non-linear systems • operations on functions • mathematical reasoning • exponential and logarithmic functions • conics • combinations • trigonometric functions

If you want to transfer from the Applied Mathematics 10–20–30 sequence to the Pure Mathematics 10–20–30 sequence at a future time, you won't have to repeat the topics that are common to pure mathematics and applied mathematics.

If you decide to transfer to Pure Mathematics 20 after successfully completing Applied Mathematics 10, you may have to take a three-credit course called Pure Mathematics 10b. If you decide to transfer to Pure Mathematics 30 after successfully completing Applied Mathematics 20, you may have to take a five-credit course called Pure Mathematics 20b. The two bridging courses are shown in the following diagram.



STRATEGIES FOR COMPLETING APPLIED MATHEMATICS 20



For each module in Applied Mathematics 20, there is a Student Module Booklet, an accompanying Assignment Booklet, and a Project Booklet. The document you are presently reading is called a Student Module Booklet.

Each Student Module Booklet will show you, step by step, what to do and how to do it. There are readings, questions for you to answer in your mathematics binder, and applications that will give you hands-on experience.

It is important to work systematically and carefully through the Student Module Booklets. This work will prepare you for the assignments, projects, and final test.

Following are some suggestions for organizing your mathematics binder:

- Keep a section of your binder to record your responses to the questions in the Student Module Booklet. Also store your marked assignments here.

- Keep a section of your binder for work in progress on your projects. Keep your research notes, plans, rough drafts, and so on.

- Keep a section of your binder to record new skills and concepts, as well as important results and formulas. Get in the habit of describing new skills and concepts in your own words.

  Record useful ways to help you remember what a concept means. Make charts and diagrams to help you connect mathematical ideas.

- Keep a section of your binder to record mathematical accomplishments. This can include solutions to problems that you are proud of solving. It can also include landmark events, such as when you grasped a difficult concept (an "aha!" experience), or when you used a calculator or spreadsheet in a new way.

Mathematical Processes

Throughout this course, you will be expected to perform the following mathematical processes:

- Connect mathematical ideas to everyday experiences and to concepts in other disciplines.
- Develop and use problem-solving strategies.
- Reason and justify your answers.
- Communicate mathematical ideas.
- Select and use appropriate technologies to solve problems.
- Develop and use estimation and mental-math strategies.
- Use visualization to assist in processing information, making connections, and solving problems.

In order to develop these mathematical processes more fully, you are encouraged to ask someone who is also taking Applied Mathematics 20 to be your study partner. You will find that having a friend to discuss mathematical ideas with will make your studying more enjoyable.

Resources You Will Need

In addition to the distance learning materials for Applied Mathematics 20, you will need the following resources:

- the *Addison-Wesley Applied Mathematics 11 Source Book*, Western Canadian Edition, published by Addison Wesley Longman Ltd. (2000)
- a binder, lined loose-leaf paper, graph paper, dividers, pencils, eraser
- metric and imperial measuring devices, such as a ruler, yardstick, metre-stick, and tape measure
- a mathematical instrument set (compass, protractor, and triangles)
- a computer with a spreadsheet program

  Note: Two popular spreadsheet programs are *ClarisWorks*™ and Microsoft® *Excel.*
- a graphing calculator

  Note: Where it is applicable, the examples in this course and the textbook show the TI-83 calculator; however, all of the graphing calculators in the following chart are approved for use on tests.

Texas Instruments	Sharp	Casio	Hewlett-Packard
TI-82** TI-83 TI-83 Plus TI-86 TI-89 TI-92* TI-92 Plus	EL-9600C EL-9600* EL-9200* EL-9300*	Algebra FX 2.0 CFX-9850 GA-Plus* CFX-9850 G* CFX-9800 G* FX-9700 series*	HP 39g

*no longer commercially available

**The TI-82 calculator will remain on the approved list for the 2000–2001 and 2001–2002 school years and will then be deleted from the approved list.

If you intend to use the TI-83 or TI-83 Plus graphing calculator, it is recommended that you obtain the video program *The TI-83 Graphing Calculator Video Tutor.*

Many of the resources that you will need may be purchased locally or from the Learning Resources Centre (LRC). Following is the LRC website:

http://www.lrc.learning.gov.ab.ca

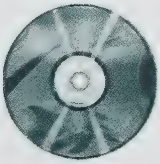
You may wish to discuss the availability of resources with your teacher, as your school division may have a loan policy.

Visual Cues

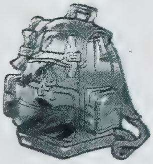
You will find many visual cues in this course. Colour is used to highlight terms that are defined in the Glossary of the Appendix of each Student Module Booklet. You will also find several icons in the margins. Read the following explanations to discover what the various icons prompt you to do.

- Refer to the textbook.

- Refer to the Applied Mathematics 20 CD.

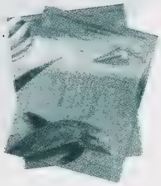


- Use mathematical instruments or measuring tools.

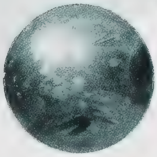


- Work with a computer.

- Complete the module project or assignment.



- Explore the Internet.



Remember: Any Internet website address given in this module is subject to change.

MODULE OVERVIEW

Have you ever wondered how a complicated piece of equipment, like an automobile, with thousands of parts built by hundreds of different people actually fits together and works? The reason is precision. As long as each part is manufactured within specifications, the pieces should be able to fit together and, thus, the assembly of the vehicle can begin. Over time, the manufacturing of these parts has become increasingly precise, making parts fit more snugly and, thus, reducing outside noises (like rattles and wind noise) and greatly improving the overall handling of today's automobile. Can you imagine how tomorrow's automobile will handle?

In this module you will study measurement and ways of describing errors in measurement. You will calculate the error in your measurements, either in absolute terms or as percentages. You will come to understand how designers specify parts—parts with a small tolerance for error so the manufacturer can be assured that these parts will fit when the finished product is assembled. You will also create orthographic drawings of three-dimensional objects—drawings much like those made by automotive designers to describe their new vehicles.

As you work through this module, you will come to appreciate that the measurements you report and the calculations you do with your measurements can appear to be much more accurate than they really are. You will write answers using a system of significant digits or using a given precision. These are skills that will serve you well if you choose to study the sciences or technologies.

ASSESSMENT

Accompanying this Student Module Booklet is a Project Booklet and an Assignment Booklet. Your grading in this module will be based upon the module project and the module assignment you submit for evaluation.

The mark distribution is as follows:

Module Project	**40 marks**
Module Assignment	**60 marks**
TOTAL	**100 marks**

Remember that Activities 1 to 6 in this Student Module Booklet will prepare you for completing the module project and the module assignment. You should work through these activities carefully and compare your answers with the suggested answers provided in the Appendix.

The Follow-up Activities provide extra help and enrichment. You may choose to do some or all the questions in the Follow-up Activities. Again, you should compare your answers with the suggested answers provided in the Appendix.

MODULE PROJECT

BUILDING A GIANT REPLICA

Beginning the Project

The giant spoon and cherry sculpture is a striking, eye-catching replica of a normal-sized object. Many small towns have built (or are building) giant replicas to gain recognition in the tourist industry. Examples in Alberta include the well-known Easter egg in Vegreville, the fork and perogy in Glendon, the ring of sausage in Mundare, the mallard ducks in Andrew, and the tepee in Medicine Hat.

Giant replicas of normal-sized objects are often required in the movie and theatre industries, too. Turn to page 356 of the textbook and read "Giant Set Design." Once you have read the page, respond to the questions posed. Store your responses in the project section of your mathematics binder.

After you have recorded some of your initial ideas for your giant replica, begin researching some of the things you will need to understand to build your giant object.

You may wish to begin your research by visiting the Internet site described on page 357 of the textbook. **Note:** The topics are under the section titled "An Ideal Home," not "Giant Set Design." These topics provide information about things relating to your project. Be sure to keep notes about your findings in the project section of your mathematics binder.

You may also enter the words *theatre set design* into one of the Internet's search engines. Here is a good website regarding theatre set design:

http://www.siue.edu/ITDA/y.html

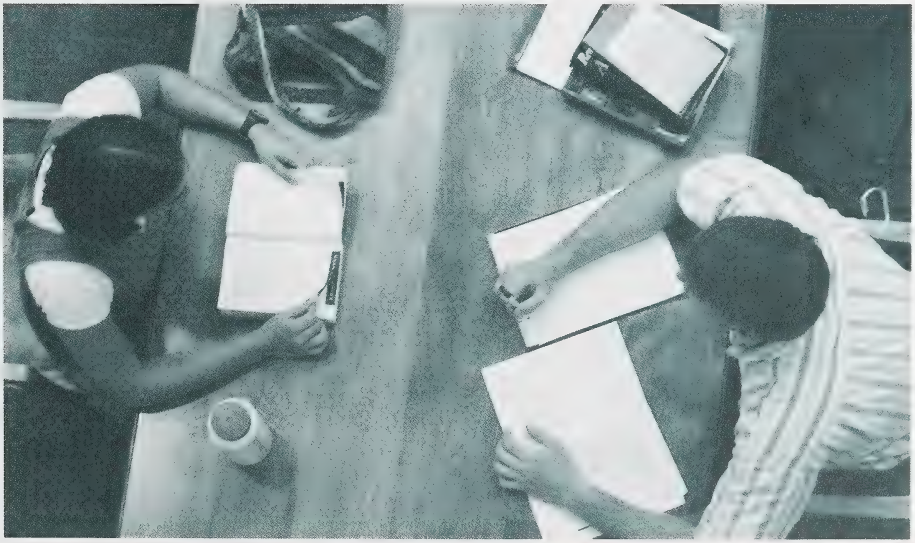
As you work through Activities 1 to 6, continue your research. Continue using the Internet; or visit your local library, researching books and magazines; or interview a person who works with measurements and the uncertainty of measurements on a daily basis (like an architect or engineer). Keep track of the resources you use in your research. You will need to add them to the bibliography (a required element) in your module project.

Since making a larger-than-life model of an object requires materials, you should give considerable thought as to what materials you have available to you. You might want to use materials like papier-mâché or play dough. Recipes for these can be found at the following sites:

- **http://venus.spaceports.com/~jrjeff/playdough.htm**
- **http://www.billnkaz.demon.co.uk/playdo.htm**
- **http://familycrafts.about.com/parenting/familycrafts/library/projects/blpmpastes.htm**

Working through Activities 1 to 6 will help you gain the skills and concepts you will need to complete the module project and the module assignment. You will complete the Building a Giant Replica project in the Project Booklet after the Follow-up Activities.

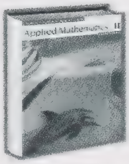
You will be given more direction on how to complete this project later in this module. In the meantime, feel free to discuss this project with a study partner or a family member. Remember, the work on the project you submit must be your own.



ACTIVITY 1

Two-Dimensional Enlargement and Reduction

Have you had a photograph enlarged? Most standard prints (measured in inches) are 4 by 6. If you want a 5 by 10 or an 11 by 14, you have to enlarge the picture. How big was the negative that produced the original print? Enlarging a photograph is one example of changing the size of a **two-dimensional** object. In this activity you will be working with two-dimensional **enlargements** and **reductions**.

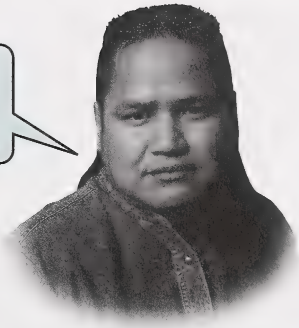


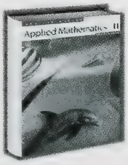
Turn to page 358 of the textbook and read the introductory paragraph of Tutorial 7.1, “Enlarging and Reducing in Two Dimensions.”

1. Answer exercises 1, 2, 4, and 5 of “Practise Your Prior Skills” on page 358 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 47–48.

You may be familiar with some scaled items, like maps and photocopies. Scale drawings are replicas of the original either enlarged or reduced.





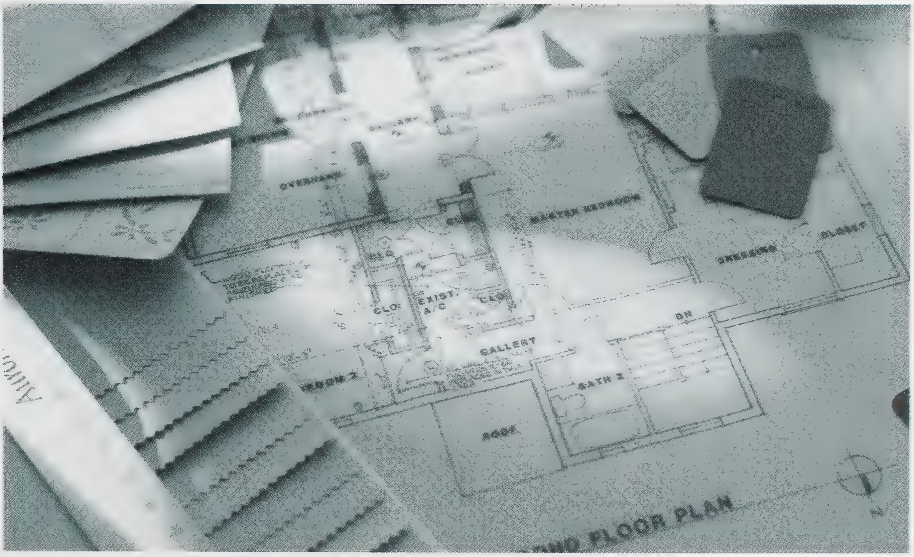
The scale on a map, for example, can be specified in several different ways. Turn to pages 359 and 360 of the textbook and read about the four different ways of representing scale on a map. Pay particular attention to the paragraph preceding Investigation 1 on page 360.

Remember: There are four main ways of specifying a scale: statement, rate, ratio, and linear scale.

2. Answer exercises 1 to 7 of "Investigation 1: Using a Map Scale" on page 360 of the textbook.

3. Answer exercises 1 to 7 of "Investigation 2: Creating a Floor Plan" on page 361 of the textbook. **Note:** You will need someone to help you make the measurements.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 48–51.



Investigation 1 allowed you to practise working with the four ways of specifying a scale. Investigation 2 provided a more practical experience of using scale factors. Now, turn to pages 361 and 362 of the textbook and work through "Example 1: Reduce an Object According to a Given Scale."

4. Answer exercises 1 to 6 of "Discussing the Ideas" on page 362 of the textbook.

5. Answer exercises 1.a., 2, 3, 5, 7, and 9 of "Exercises: Checking Your Skills" on pages 363 to 365 of the textbook.

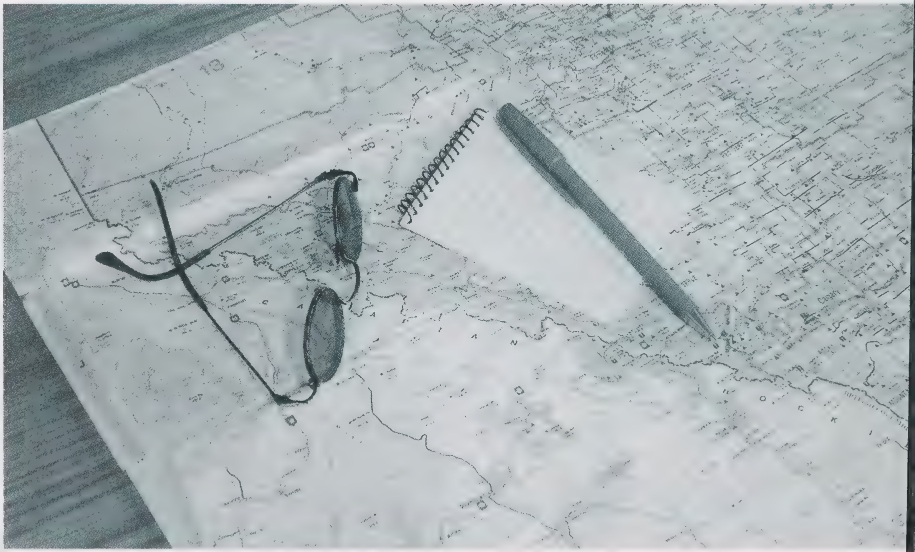
Compare your responses with the suggested answers in the Appendix, Activity 1, pages 51–55.

LOOKING BACK

In this activity you worked with scales and scale drawings. You were shown that there are four different ways of specifying scales. It would be beneficial for you to remember these as you work through this module.

6. Turn to page 365 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 1, page 55.



ACTIVITY 2

Portraying Three-Dimensional Objects

You live in a three-dimensional world. Your eyes are designed to give you the perception of depth so you function well in this environment. Drawing tools and materials, however, are generally two-dimensional. It took millennia to devise ways of showing the three-dimensional characteristics of objects (like the building in the photograph) on a two-dimensional piece of paper. Those interested in art will already know about perspective and how to use it to make a drawing seem to have depth.

In this activity you will study other ways of showing three-dimensional objects on a two-dimensional piece of paper.

Turn to page 366 of the textbook and read the introductory paragraphs of Tutorial 7.2, “Diagrams of Three-Dimensional Objects.” Make sure you understand how internal features and different levels are shown in an **orthographic drawing**.

You can think of an orthographic drawing as an object that is placed in a rectangular, glass box. First, the object is traced onto each side of the box; then the box is unfolded, and the object is removed. The resulting views, six altogether, form an orthographic diagram.

Turn to pages 367 and 368 of the textbook and work through "Example 1: Draw an Orthographic Diagram of an Object with Holes Through It" and "Example 2: Draw an Orthographic Diagram of an Object with a Circular Cross-Section." Pay particular attention to how the different views represent different ways of looking at the object. **Note:** In Example 1, notice how a circular hole in reality shows up as an ellipse in the pictorial representation. This is one of several things that makes visualizing three-dimensional objects difficult.

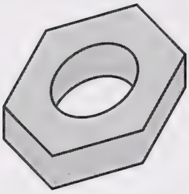
Now, turn to pages 368 and 369 of the textbook and work through "Example 3: Decreasing the Size of a Machine Part." You will be using information in the diagrams to find facts about the real object.

1. Answer exercises 1 to 3 of "Discussing the Ideas" on page 369 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 2, page 56.

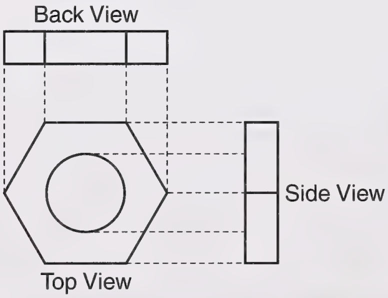
Example

Here is an example of taking an object in pictorial form and turning it into several orthographic views. The object is a hexagonal nut.



Solution

You know that an ellipse in pictorial form is often the result of tilting a circle; so the object most likely has a circular hole through the centre. (A circular hole is what you would expect with a nut anyway.) It will also have six equal sides. The orthographic drawing is as follows; it includes a top view, back view, and side view.

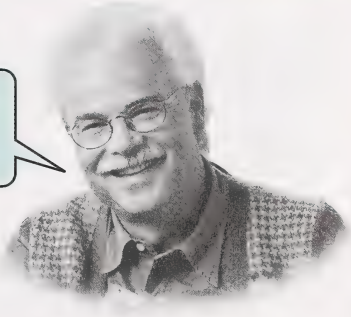


2. Turn to pages 370 and 371 of the textbook and do exercises 1.a., 2.a., 3, 4, and 7 of “Exercises: Checking Your Skills.”

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 56–58.

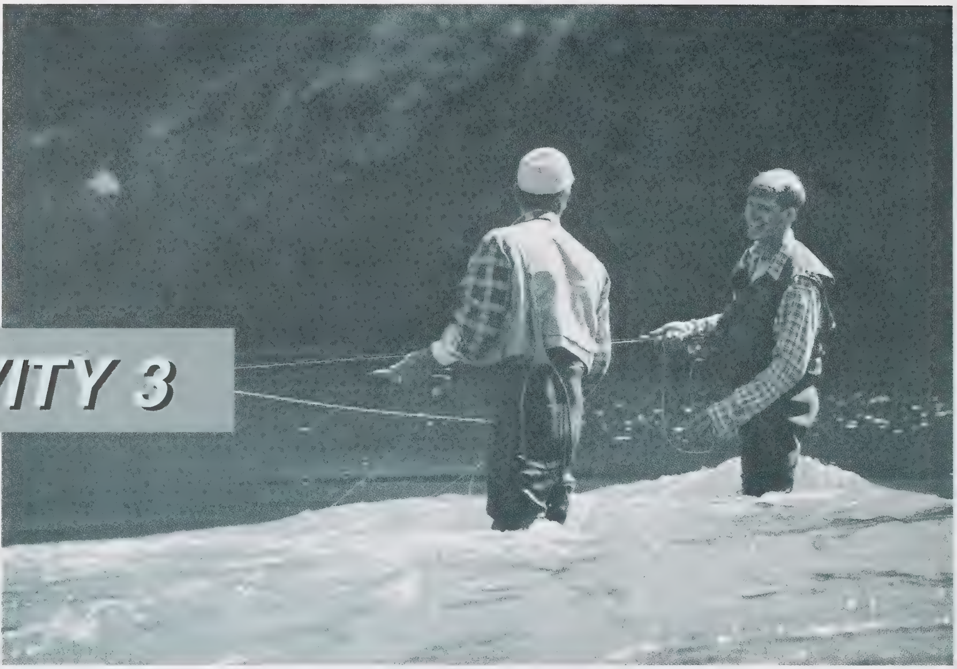
LOOKING BACK

In this activity you defined pictorial and orthographic diagrams of three-dimensional objects. You then drew orthographic diagrams of these objects.



3. Turn to page 371 of the textbook and answer “Communicating the Ideas”.

Compare your response with the suggested answer in the Appendix, Activity 2, page 58.



ACTIVITY 3

Accuracy in Measurement

Have you or one of your friends ever told a story about the fish that got away? Quite often one story brings on other stories of even bigger fish that got away. Listening in, a friend might wonder whether the measurements talked about are true or, possibly, a little exaggerated. In this activity you will study how to specify the uncertainty in measurements. Was that fish really three feet long, or was it three feet long plus or minus two feet?

Turn to page 372 of the textbook and read the introductory paragraphs of Tutorial 7.3, "The Uncertainty of a Measurement." The introduction extends to the blue rectangle part way down the page.

Thus far, you have studied **precision** and **accuracy** in earlier mathematics courses. Now, you are going to build on this knowledge and learn a way to express the possible errors that occur when you measure objects.

Turn to pages 372 and 373 of the textbook and continue reading the discussion of uncertainty of measurement.

1. What is uncertainty of measurement?

2. What is the precision of a measuring instrument?

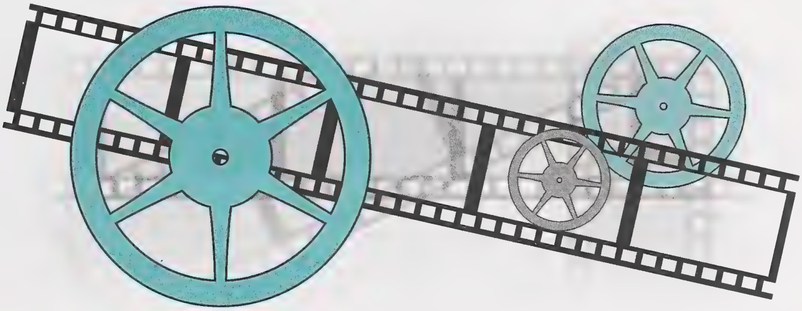
3. Why has the zero been added to the measurement of the bolt on page 373?

Compare your responses with the suggested answers in the Appendix, Activity 3, page 59.

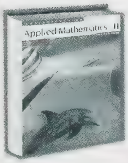
REMEMBER

1. *The uncertainty of a measurement, if not given, is one-half of the precision of the measurement.*
2. *Measurements are recorded with the same number of decimal places as the uncertainty.*
3. *Precision is equal to the smallest scale division on the measuring instrument.*

For more information on precision and accuracy, view the segment titled *Precision and Accuracy* on the Applied Mathematics 20 CD.



So far, you have defined precision and uncertainty when making a measurement. Now, you will analyse the affect adding or subtracting measurements has on the uncertainty of the result.



4. Turn to page 374 of the textbook and answer exercises 1 to 10 of “Investigation 1: Adding and Subtracting Measurements with the Same Precision.”

5. Turn to page 375 and read the paragraph at the top of the page; then answer exercises 1 to 4 of “Investigation 2: Adding and Subtracting Measurements with Different Precision.”

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 59–61.

The following concluding statements can be made for Investigations 1 and 2.

Concluding Statement 1

The uncertainty in a sum or difference of measurements is the sum of the uncertainties of the individual measurements.

Concluding Statement 2

When adding or subtracting measurements, the precision of the reported result equals the precision of the least-precise measurement and the uncertainty is reported with the precision of the least-precise uncertainty.

The next example reinforces the meaning of Concluding Statement 1 and Concluding Statement 2.

Example

Following is a set of measurements and their uncertainties:

$$24.80 \pm 0.05 \text{ cm},\ 14.0 \pm 0.5 \text{ cm},\ 12.80 \pm 0.05 \text{ cm},\ 20 \pm 1 \text{ cm}$$

a. What is the uncertainty of the sum of these measurements?

b. What is the sum of these measurements?

Solution

a. The uncertainty of a sum of measurements is the sum of the uncertainties of the individual measurements. The uncertainties are 0.05 cm, 0.5 cm, 0.05 cm, and 1 cm. The sum of these uncertainties is

$$0.05 + 0.5 + 0.05 + 1 = 1.6 \text{ cm}$$

b. $24.80 + 14.0 + 12.80 + 20 = 71.60$

The sum of the measurements is 71.60 cm. Since the precision and uncertainty of a sum is the precision and uncertainty of the least-precise measurement, the sum is (72 ± 2) cm. Here, 71.60 is rounded to the nearest centimetre (the same precision as the least-precise measurement). The uncertainty is rounded up to the nearest centimetre (the same precision as the least-precise uncertainty).

When rounding uncertainties, always round up (e.g., 1.2 becomes 2 as does 1.1 and 1.8). This ensures that the maximum and minimum values will be inside the uncertainty interval.

Turn to pages 376 and 377 of the textbook and work through "Example 1: Adding Measurements" and "Example 2: Subtracting Measurements."

As you can see, error in measurements can cause problems in real-world settings. Just knowing that the garage, for example, measures 11.3 m doesn't guarantee that the two buses, whose lengths have a sum of 11.3 m, will fit into it.

6. Answer exercises 1 to 5 of "Discussing the Ideas" on page 377 of the textbook. These exercises will help clarify the ideas being discussed in this activity.

7. **a.** Answer exercises 1, 2, 3, 5, and 6 of "Exercises: Checking Your Skills" on pages 378 and 379 of the textbook.

b. Answer exercise 8 of "Exercises: Extending Your Thinking" on page 379 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 61–65.

The project in this module is to build an enlarged replica of a real-world object. Turn to page 380 of the textbook and work through the bulleted items. Save your work for your module project.

LOOKING BACK

In this activity you added and subtracted measurements. You found that the uncertainties must be added when adding or subtracting measurements. You also determined that the precision of a sum or difference is equal to that of the least-precise measurement and that the uncertainty has the precision of the least-precise uncertainty.

8. Turn to page 379 of the textbook and answer "Communicating the Ideas."

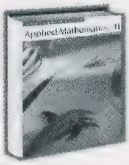
Compare your response with the suggested answer in the Appendix, Activity 3, page 65.

ACTIVITY 4

Using Percentage to Describe Error

Errors in medical or industrial tests can be costly in both money and lives. The workers in this laboratory are doing their best to minimize the errors in their work. Yet, they know that error is a part of all of the test results they complete. Often, results in scientific tests are given in percentage form so errors in different tests can be readily compared.

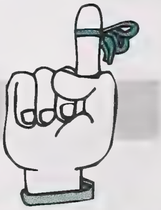
In this activity you will determine how to calculate percentage error.



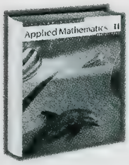
Turn to page 382 of the textbook and read the introductory paragraph of Tutorial 7.4, "Percentage Error."

1. Answer exercises 1 to 4 of "Practise Your Prior Skills" on page 382 of the textbook.

Remember: Percent means "out of 100."



Compare your responses with the suggested answers in the Appendix, Activity 4, pages 65–66.



Turn to pages 382 and 383 of the textbook and work through “Example 1: Comparing Accuracy.”

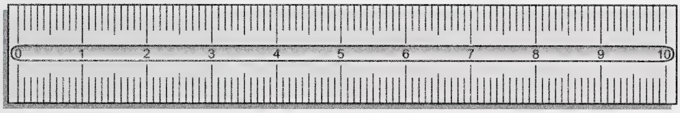
2. What other factors might this factory owner consider before making a purchasing decision?

Compare your response with the suggested answer in the Appendix, Activity 4, page 66.

Now, turn to page 383 of the textbook and read the discussion on percentage error. Then turn to page 384 and work through “Example 2: Comparing Measurements.”

Example

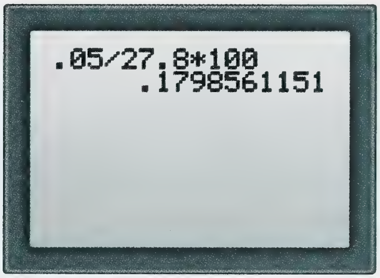
What is the percentage error for a measurement of (27.8 ± 0.05) cm?



Solution

Percentage error is determined by converting the ratio of uncertainty of measurement to reported measurement into a percentage. In this case, the uncertainty is 0.05 cm and the measurement is 27.8 cm.

$$
\begin{aligned}
\therefore \text{ Percentage error} &= \frac{\text{Uncertainty}}{\text{Measurement}} \times 100\% \\
&= \frac{0.05}{27.8} \times 100\% \\
&\doteq 0.179\,856\,115\,1\%
\end{aligned}
$$



```
.05/27.8*100
          .1798561151
```

When doing calculations with your calculator, do not round any of the results. Only round calculated values when you are reporting the final answer.

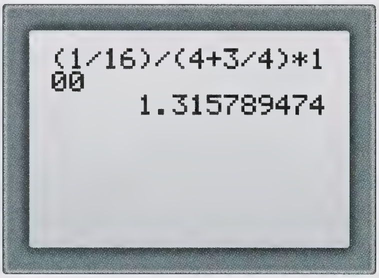
The percentage error is about 0.18%.

Percentage error varies with the size of the object being measured. It is, therefore, also called relative error. For information on comparing the percentage error of objects of different sizes, view the segment titled *Percentage Error* on the Applied Mathematics 20 CD.

Now, turn to page 384 of the textbook and work through "Example 3: Calculating Percentage Error for Imperial Measurement." Pay particular attention to the hints on using the calculator and rounding.

You could use your calculator the following way to find the percentage error in Example 3. Notice the use of addition to enter the mixed number $4\frac{3}{4}$ into the calculator.

$$\frac{\frac{1}{16}}{4\frac{3}{4}} \times 100\% \doteq 1.32\%$$



```
(1/16)/(4+3/4)*1
00
       1.315789474
```

3. Turn to page 385 of the textbook and answer exercises 1 to 4 of "Discussing the Ideas."

4. Turn to pages 385 and 386 of the textbook and answer the following. Round the percentage error to the same number of decimal places as there are in the uncertainty.

   a. exercises 1, 2, 3, 5, and 6 of "Exercises: Checking Your Skills"
   b. exercise 8 of "Exercises: Extending Your Thinking"

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 66–70.

LOOKING BACK

In this activity you examined and calculated percentage error. You then used these calculations to compare the accuracy of different measurements.

5. Turn to page 386 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 4, page 70.

ACTIVITY 5

Describing Error in Products and Quotients

When you are building a concrete driveway or pouring a foundation, whether for a house or office building, you have to know how much concrete to order. Concrete is often ordered by the cubic yard. To complete a 50-foot by 15-foot by 5-inch driveway, how do you determine how much concrete to order? You do this by converting each measurement into yards and then multiply. The result is the number of cubic yards of concrete you will need. How do you determine the uncertainty of your calculation, though? After all, if you order too much concrete, what do you do with it? Too little wouldn't be very good either.

In this activity you will investigate the rules for determining percentage error when multiplying and dividing measurements.

Turn to page 387 of the textbook and work through "Example 1: Determine the Percentage Error in a Product of Measurements."

In Example 1, the greatest and least possible lengths and widths are determined by adding or subtracting the amount of error from each measurement. The amount of error is calculated by multiplying each measurement by the percentage errror as a decimal.

$$\begin{aligned}\text{Error in length} &= 43 \times 0.025 \\ &= 1.075 \text{ m}\end{aligned}$$

$$\begin{aligned}\therefore \text{ Greatest possible length} &= 43 + 1.075 \\ &= 44.075 \text{ m}\end{aligned}$$

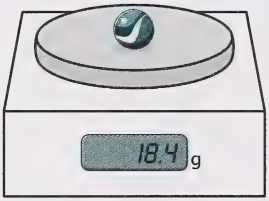
$$\begin{aligned}\therefore \text{ Least possible length} &= 43 - 1.075 \\ &= 41.925 \text{ m}\end{aligned}$$

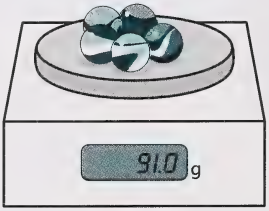
The calculations for the greatest and least possible widths are similar.

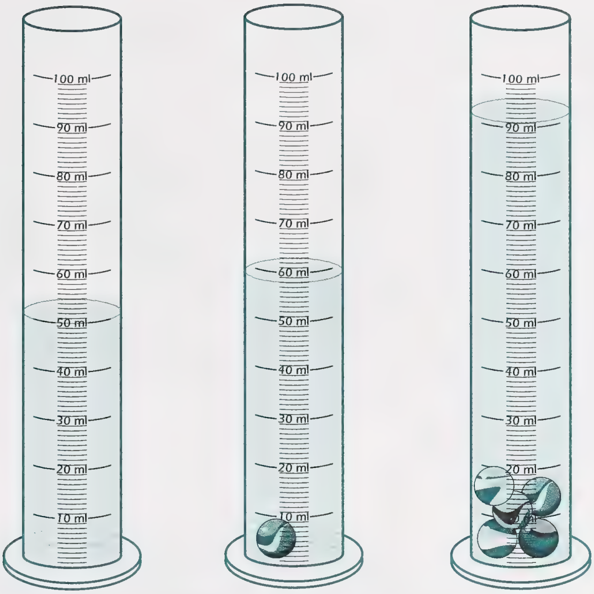
Now, read the rule given at the bottom of page 387 of the textbook regarding calculating the percentage error of a product of measurements.

Remember: The percentage error of a product is the sum of the percentage errors of the individual measurements (when the uncertainties are small).

1. Turn to pages 390 to 392 of the textbook and answer the following.
   - **a.** exercise 2 of "Discussing the Ideas"
   - **b.** exercises 1, 2, 3, and 5 of "Exercises: Checking Your Skills"
2. Turn to page 388 of the textbook and read the introduction to "Investigation: The Percentage Error of the Quotient of Two Measurements." Then do exercises 1 to 10. If you have the materials to carry out the investigation, do so. If you do not have the materials, use the following diagrams to complete exercises 1 to 10.

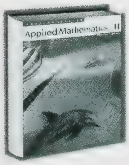






Compare your responses with the suggested answers in the Appendix, Activity 5, pages 70–76.

Remember: The percentage error of a quotient is the sum of the percentage errors of the individual measurements (when the uncertainties are small).



3. Turn to pages 390 to 393 of the textbook and answer the following.

- **a.** exercises 1 and 5 of “Discussing the Ideas”
- **b.** exercises 6 and 8 of “Exercises: Checking Your Skills”
- **c.** exercise 9 of “Exercises: Extending Your Thinking”

Compare your responses with the suggested answers in the Appendix, Activity 5, pages 77–80.

Turn to page 389 of the textbook and read the discussion regarding significant digits at the top of the page.

Significant digits are used heavily in the sciences. The trick is to determine which is the first uncertain digit.

Turn to page 389 of the textbook and work through "Example 2: Determine the Number of Significant Digits." You will come away with a much better idea of significant digits.

Notice that all non-zero digits are significant. With zero digits, though, you must figure out when they are significant. For example, the zero in 309 is significant, but not in 390.

Example

How many significant digits are there in the following values, assuming they were obtained by measuring real-world objects?

a. 10.0 cm
b. 100 m
c. 0.105 m
d. 0.015 m
e. 0.010
f. 10 cm
g. 101 m
h. (400 ± 10) m

Solution

a. 10.0 cm has 3 significant digits. The zeros following a decimal place and to the right of a non-zero digit are significant.

b. 100 m has 1 significant digit. The zeros to the right of a non-decimal number are not significant.

c. 0.105 m has 3 significant digits. The zeros between non-zero digits are significant.

d. 0.015 m has 2 significant digits. The zeros to the left of the first non-zero digit are not significant.

e. 0.010 m has 2 significant digits. The zeros following a decimal place and to the right of a non-zero digit are significant.

f. 10 cm has 1 significant digit. The zeros at the right of a non-decimal number are not significant.

g. 101 m has 3 significant digits. The zeros between non-zero digits are significant.

h. (400 ± 10) m has 2 significant digits. The tens digit is the first uncertain digit.

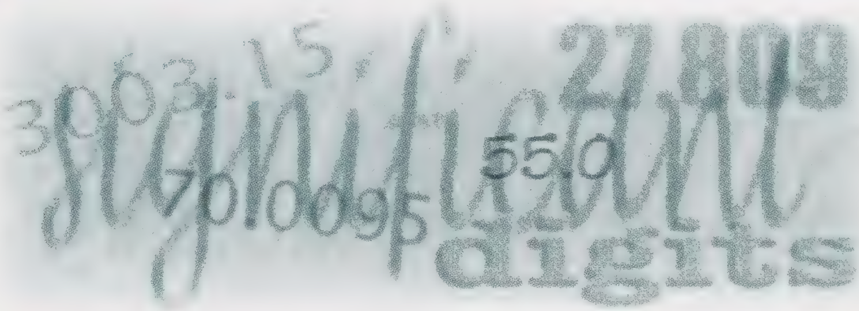
Turn to page 390 of the textbook and read the information at the top of the page regarding significant digits as they apply to products and quotients of measurements.

4. Answer exercises 3 and 4 of "Discussing the Ideas" on page 390 of the textbook.

5. State the number of significant digits in the following measurements.

a. 93.0
b. 4000
c. (600 ± 1) gm
d. 0.0087
e. 12.30
f. 0.0320
g. $(51\,000 \pm 100)$ m

Compare your responses with the suggested answers in the Appendix, Activity 5, page 81.



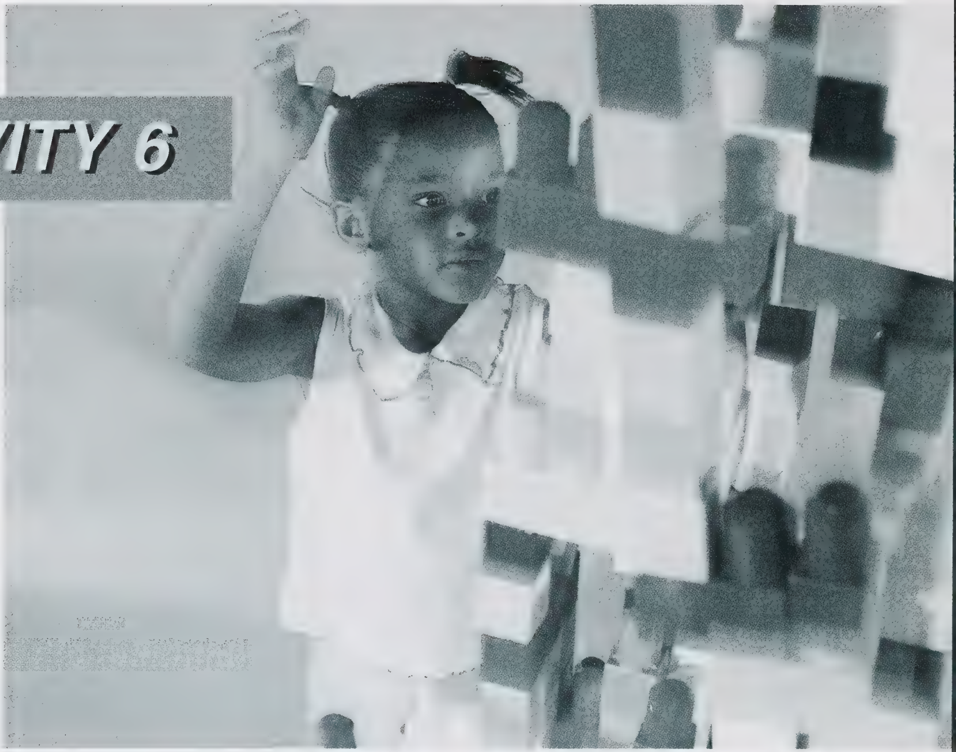
LOOKING BACK

In this activity you performed calculations with measured data. You also applied the idea of significant digits to reporting answers.

6. Turn to page 393 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 5, page 81.

ACTIVITY 6

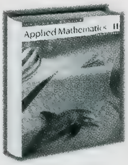


How Close Is Close Enough?

Building blocks are designed to fit easily, yet snugly. They have been built to ensure that youngsters who are playing with them don't have trouble fitting the pieces together. There may be slight variations in the sizes of the pieces, but not large enough to frustrate the user. You might say that these mass-produced parts have been created with a small tolerance for size differences.

In this activity you will be working with tolerances and how they affect most parts of society today.

Precise measurements are a requirement in many industrial applications. Before you begin your study of tolerance, view the segment titled *Measurement* on the Applied Mathematics 20 CD. This segment will give you an overview of a number of measuring instruments with various degrees of precision.



Turn to page 394 of the textbook and read the introductory paragraphs of Tutorial 7.6, “Applying Tolerances.” Pay particular attention to the two new terms defined: **tolerance range** and **tolerance interval**. Then work through “Example 1: Testing Bulletproof Vests” and “Example 2: Maintaining Building Codes” on pages 394 to 396. These examples give you a chance to see the two new terms used in a couple of different situations.

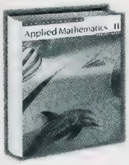
Specifications for materials are prepared to ensure that they will work with other parts, that they will work as expected, and that they will work safely.

1. The specifications for a certain part consist of a length of (12.0 ± 0.1) cm. How long would the part have to be before it is too long for this specification? How short would it have to be before it is too short?

Compare your response with the suggested answer in the Appendix, Activity 6, page 82.

The next example requires the use of a formula rather than a simple use of the given tolerances.

Example



Turn to page 396 of the textbook and read the problem for “Example 3: Race Car Engine Requirements.”

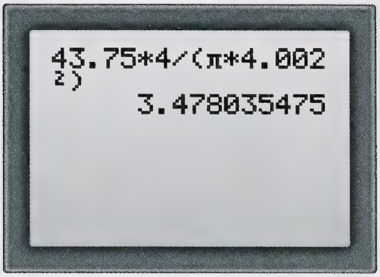
The textbook solution to this problem is incorrect. If you check the maximum height of 3.485 inches with a maximum diameter of 4.002 inches, you will find that the volume of 350 cubic inches for 8 cylinders is slightly exceeded. Two corrected solutions to this interesting problem are given here.

Solution

Method 1

The maximum allowable volume is 350 cubic inches. This volume is calculated from the maximum diameter and the maximum height using the equation $V = \frac{\pi d^2 h}{4}$ for each of the eight cylinders. The maximum volume for one cylinder is then $\frac{350}{8} = 43.75$ cubic inches. The maximum diameter is 4.002 inches. Substituting the known maximum values into the equation for the volume of a cylinder will determine the maximum acceptable height.

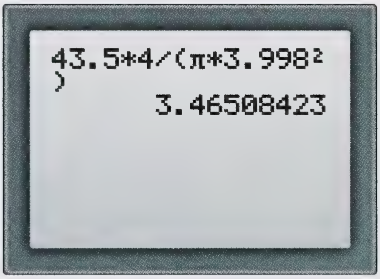
$$
\begin{aligned}
V &= \frac{\pi d^2 h}{4} \\
43.75 &= \frac{\pi (4.002)^2 \ h}{4} \\
h &= \frac{43.75(4)}{\pi (4.002)^2} \\
&\doteq 3.478\,035\,475 \text{ inches}
\end{aligned}
$$



```
43.75*4/(π*4.002
²)
                3.478035475
```

The minimum allowable volume is 348 cubic inches. This volume is calculated from the minimum diameter and the minimum height using the equation $V = \frac{\pi d^2 h}{4}$ for each of the eight cylinders. The minimum volume for one cylinder is then $\frac{348}{8} = 43.5$ cubic inches. The minimum diameter is 3.998 inches. Substituting the known minimum values into the equation for the volume of a cylinder will determine the minimum acceptable height.

$$
\begin{aligned}
V &= \frac{\pi d^2 h}{4} \\
43.5 &= \frac{\pi (3.998)^2 \ h}{4} \\
h &= \frac{43.5(4)}{\pi (3.998)^2} \\
&\doteq 3.465\,084\,23 \text{ inches}
\end{aligned}
$$



```
43.5*4/(π*3.998²
)
                 3.46508423
```

Half of the difference of the maximum and minimum values is the desired tolerance.

$$
\begin{aligned}
\therefore \text{ Tolerance} &\doteq \frac{3.478\,035\,475 - 3.465\,084\,23}{2} \\
&\doteq 0.006\,475\,62 \text{ inches}
\end{aligned}
$$

Therefore, the tolerance interval is (3.472 ± 0.006) inches.

Note: For tolerance calculations, you should round down to ensure that values that are too large are not included in the tolerance interval.

Method 2

The person building the engine can change two things: the diameter and the height of each cylinder. These are the inputs to the equation $V = \frac{\pi d^2 h}{4}$, which is used to calculate the volume. The nominal height can be calculated from $V = \frac{\pi d^2 h}{4}$, remembering that the volume of one cyliner is $\frac{1}{8}$ of the total volume.

$$
\begin{aligned}
V &= \frac{\pi d^2 h}{4} \\
\frac{349}{8} &= \frac{\pi (4.000)^2 h}{4} \\
\frac{349}{8} &= \frac{\overset{4}{\cancel{16}}\pi h}{\underset{1}{\cancel{4}}} \\
\frac{349}{8} &= 4\pi h \\
h &= \frac{349}{8(4\pi)} \\
&\doteq 3.471\,567\,196
\end{aligned}
$$

Recall that the sum of the percentage errors in a product will give the percentage error in the result.

$$
\begin{aligned}
\therefore \text{ Percentage error (volume)} &= \frac{1}{349} \times 100\% \\
&\doteq 0.286\,532\,951\,3\%
\end{aligned}
$$

$$
\begin{aligned}
\therefore \text{ Percentage error (diameter)} &= \frac{0.002}{4} \times 100\% \\
&= 0.05\%
\end{aligned}
$$

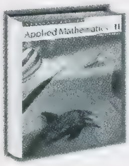
Because the volume is calculated by multiplying the height by the diameter squared, the percentage errors give the following equation:

$$
\begin{aligned}
\text{Error}_{\text{volume}} &= \text{Error}_{\text{height}} + \text{Error}_{\text{diameter}} + \text{Error}_{\text{diameter}} \\
0.286\,532\,951\,3\% &\doteq \text{Error}_{\text{height}} + 0.05\% + 0.05\% \\
\text{Error}_{\text{height}} &\doteq 0.286\,532\,951\,3\% - 0.10\% \\
\text{Error}_{\text{height}} &\doteq 0.186\,532\,951\,3\%
\end{aligned}
$$

The nominal height is 3.471 567 196 inches; this results in a tolerance for height of $3.471\,567\,196 \times 0.001\,865\,329\,513 \doteq 0.006\,475\,623\,7$ inches.

The precision of measurements is to thousandths of an inch; so, round the results to the thousandths place. The tolerance interval for the cylinder height is (3.472 ± 0.006) inches. (For tolerances, you should round down to ensure that no unacceptable values are included.) The maximum height is 3.478 inches and the minimum height is 3.466 inches.

Another error in the textbook occurs where the solution adds the percentage error of an input value (diameter) and an output value (volume) together.



2. Turn to pages 397 to 399 of the textbook and answer the following.

   a. exercises 1 to 4 of "Discussing the Ideas"
   b. exercises 1 to 4 of "Exercises: Checking Your Skills"
   c. exercises 7 and 8 of "Exercises: Extending Your Thinking"

Compare your responses with the suggested answers in the Appendix, Activity 6, pages 82–85.

Tolerance of measurement is important in various occupations. For a machinist's view on the importance of understanding tolerance and the value of mathematics, view the segment titled *Machinist* on the Applied Mathematics 20 CD.

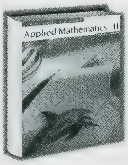
LOOKING BACK

In this activity you worked with tolerances. You discovered that tolerances uses ± notation, as with the other methods of displaying error. You also found that applications of tolerances appear in numerous industries.

3. Turn to page 399 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 6, page 86.

Follow-up Activities



This module dealt with Chapter 7: Measurement in the *Addison-Wesley Applied Mathematics 11 Source Book.*

Turn to page 402 of the textbook and review the skills and concepts that were developed in this module. Also, read the column of important results and formulas you discovered.

Now, answer exercises 2, 3.a., 4, 5, 6, 8, 9, and 10 of Part B of "What Should I Be Able to Do?" on pages 404 and 405 of the textbook.

Compare your responses with the suggested answers in the Appendix, Follow-up Activities, pages 86–88.

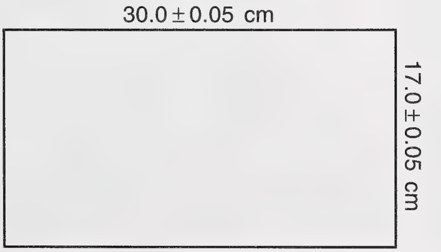
If you had difficulties understanding the skills and concepts in Module 7: Measurement, it is recommended that you do the Extra Help. If you have a clear understanding of the skills and concepts in this module, it is recommended that you do the Enrichment. You may decide to do both.

EXTRA HELP

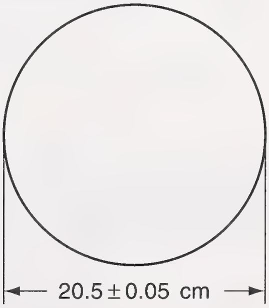
In this module, you explored measurements, ways to describe three-dimensional objects, and methods of reporting errors in measurement. When reporting the answers to questions involving measured values, you need to consider the precision of the measurements and the number of significant digits in the measurements before giving your answer. You do not want to mislead anyone by giving an answer that appears to be more precise than your measurements allow.

Look at the following diagrams and the measured values given. Report the area of each object using ± notation for amount of error and for percentage error.

1.



2.



Compare your responses with the suggested answers in the Appendix, Extra Help, pages 88–89.

For a comparison of a measurement using different measuring devices, view the segment titled *Precision: Floppy Disk* on the Applied Mathematics 20 CD.

ENRICHMENT

There are many more measuring systems than the imperial and metric systems you have used in this module. The following Internet site contains a good explanation of the need for measuring systems that are much larger in scale than the two mentioned previously.

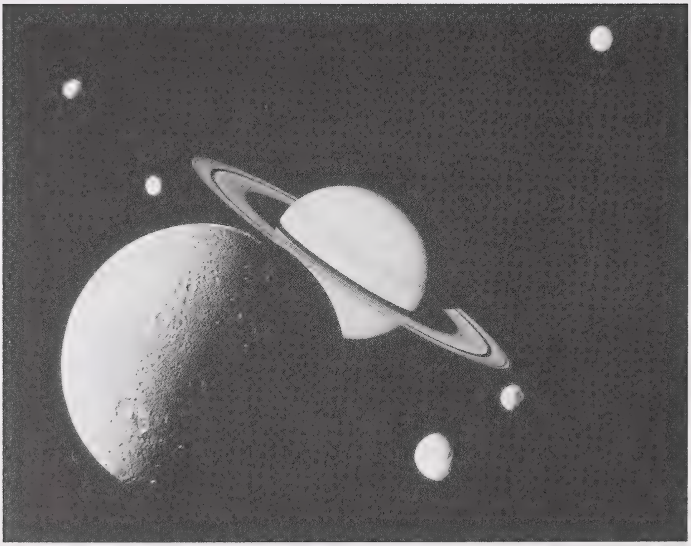
http://www.badastronomy.com/bad/misc/light_years.html

The following Internet site provides a good definition of a *light year* and why it is used to measure distance:

http://www.howstuffworks.com/question94.htm

Finally, another system for measuring distances within the solar system is the Astronomical Unit. An Astronomical Unit is the mean distance between the Earth and the Sun. It is defined at the following website:

http://www.bartleby.com/65/as/astronomU.html



Pluto is 39 AU from the Sun. Report this distance using percentage error.

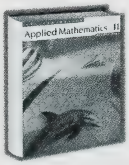
Compare your response with the suggested answer in the Appendix, Enrichment, page 90.

MODULE PROJECT

BUILDING A GIANT REPLICA

Completing the Project

By now you should have completed the initial research into your module project, Building a Giant Replica. You are to create a giant object of your own. The materials and methods you choose to use are up to you. Depending on the object, you might want to use play dough, clay, or papier-mâché. Other materials are also acceptable for building your object.



Turn to page 400 of the textbook and read "Building the Giant Replica." You will find out exactly what you need to do to complete your project.

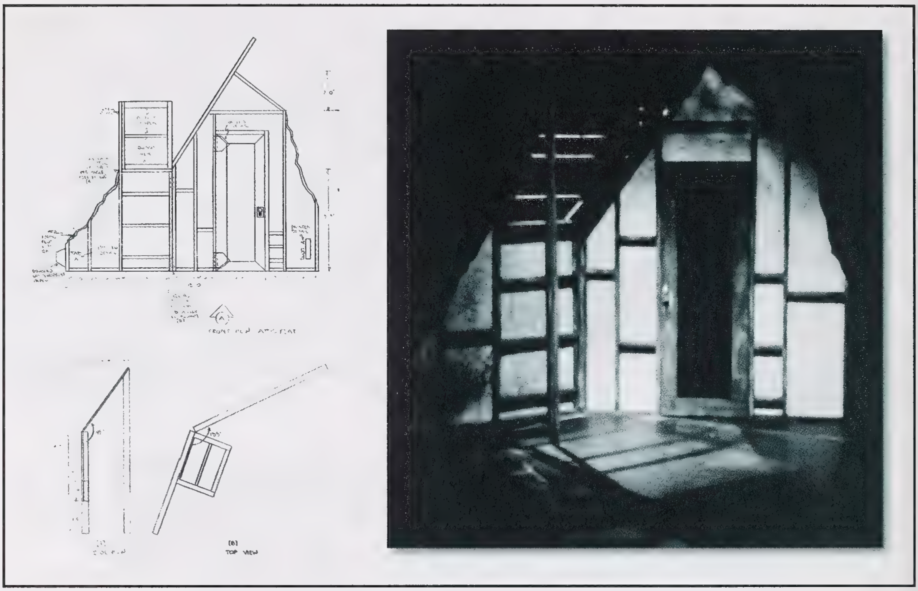
Answer exercises 11 and 12 of Part C of "What Should I Be Able to Do?" on pages 407 and 408 of the textbook.

Compare your responses with the suggested answers in the Appendix, Module Project, pages 90–91.

The preceding exercises should have helped clarify what should be included when you submit your project for marking. For this course, you may submit the giant object you have created (a pencil, a ring, a doorknob, a key, and so on) or you may submit photographs of the object.

If you choose to submit photographs, include a ruler next to the object in each photograph so the size can be judged. Also, show the object in the same views as your orthographic drawing. Finally, include another photograph of the giant object beside the original object so the scale is clear and the special features of the original and giant objects can be compared.

The following shows the orthographic diagrams and a photo of the final stage set done as a project by a student in a theatrical design program. Your designs for your module project will not be as extensive as this, but it gives you an idea of how orthographic drawings are representative of the final creation.



COURTESTY OF CATHERINE MUDRYK

The Module Project

You are to submit several items to your teacher for the module project. Having seen the kind of work shown in the sample project work on pages 406 to 408 of the textbook, you may want to revise or enhance some of your thoughts about your design.

The project will be graded in four areas, each worth 5 marks.

Sketch and Object	**5 marks**
Orthographic Drawing	**5 marks**
Giant Object (or Photographs)	**5 marks**
Report	**5 marks**
TOTAL	**20 marks**

Sketch and Object

To obtain maximum marks in this category, you must include the original object itself (or photographs of the original object, as described earlier) and a sketch showing the following:

- the original object
- significant features of the object
- the measurements of the object (e.g., 3.1 cm)
- the precision of the measurements (e.g., (3.1 ± 0.05) cm)

Orthographic Drawing

To obtain maximum marks in this category, your drawing must

- state the scale explicitly
- clearly state the measurements
- state the tolerance of each measurement
- use solid lines for all outside edges
- use broken lines for all internal features
- use broken lines to show how different views are aligned
- include all needed views
- provide a clear, concise, and complete drawing

Giant Object

To obtain maximum marks in this category, you must

- provide measurements that are correct to scale
- provide measurements that are within tolerances
- use colour where possible
- provide accurate details
- show that the giant object is constructed carefully

Report

To obtain maximum marks in this category, you must provide

- a clear description of the process used in building the model
- reasons for using the scale chosen
- detailed calculations
- calculations that are free of mathematical errors
- a bibliography

Submit your completed Module 7 Project Booklet to your teacher.

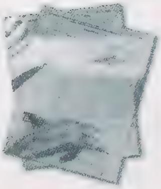
Module Summary

In this module you created orthographic drawings of three-dimensional objects and were shown different ways to express scale. You also identified different ways of describing errors in measurement and solved problems involving uncertainities in measurement.

In your studies, you determined percentage error when multiplying and dividing measurements. You discovered that by using percentage error, you can compare errors involving different measuring systems. You then examined how tolerance is expressed and how, for example, it is used to create parts that, when assembled, will fit to a specified precision.

The skills that you acquired in completing this module are used everyday by people, like design engineers, who produce plans for the manufacture of new cars. Unlike the vehicles of the early part of the century, today's vehicle calls for precisely constructed parts that fit with great accuracy. Can you think of other occupations where skills associated with precise measurement and reporting the precision of the measurement are required?

Module Assignment



To demonstrate what you have learned in this module, complete the module assignment in the Assignment Booklet.

Submit your completed Module 7 Assignment Booklet to your teacher.

COURSE SURVEY FOR APPLIED MATHEMATICS 20
(© 2001)

After you have completed the assignments in this course, please fill out this questionnaire and mail it to the address given on the last page. This course is designed in a new distance learning format, so we are interested in your responses. Your constructive comments will be greatly appreciated, as future course revisions can then incorporate any necessary improvements.

Name ____________________ Age ☐ under 19 ☐ 19 to 40 ☐ over 40

Address ____________________ File No. ____________________

____________________ Date ____________________

____

Design

1. This course contains a series of Student Module Booklets. Do you like the idea of separate booklets?

2. Have you ever enrolled in a correspondence or distance learning course that arrived as one large volume?

   ☐ Yes ☐ No If yes, which style do you prefer?

3. The Student Module Booklets contain a variety of self-assessed activities. Did you find it helpful to be able to check your work and have immediate feedback?

   ☐ Yes ☐ No If yes, explain.

4. Were the questions and directions easy to understand?

   ☐ Yes ☐ No If no, explain.

5. Each section contains follow-up activities. Which type of follow-up activity did you choose?

   ☐ mainly extra help
   ☐ mainly enrichment
   ☐ a variety
   ☐ none

   Did you find these activities beneficial?

   ☐ Yes ☐ No If no, explain.

6. Did you understand what was expected in the Assignment Booklets?

   ☐ Yes ☐ No If no, explain.

7. The course materials were designed to be completed by students working independently at a distance. Were you always aware of what you had to do?

   ☐ Yes ☐ No If no, provide details.

8. This distance learning course may include an assortment of drawings, photographs, and charts.

   a. Did you find the visuals in this course helpful?

      ☐ Yes ☐ No Comment on the lines below.

   b. Did you find the variety of visuals in this course motivating?

      ☐ Yes ☐ No Comment on the lines below.

9. Suggestions for audiocassette, videocassette, and computer activities may have been included in the course. Did you complete these media activities?

☐ Yes ☐ No Comment on the lines below.

Only students enrolled in a junior high course need to complete the following question.

10. The Student Module Booklet may have directed you to work with your teacher. How well did you work as a team?

Student's comments:

Teacher's comments:

Course Content

1. Was enough detailed information provided to help you learn the expected skills and ojectives?

☐ Yes ☐ No Comment on the lines below.

2. Did you find the workload reasonable?

☐ Yes ☐ No If no, explain.

3. Did you have any difficulty with the reading level?

☐ Yes ☐ No Please comment.

4. How would you assess your general reading level?

☐ poor reader ☐ average reader ☐ good reader

5. Was the material presented clearly and with sufficient depth?

☐ Yes ☐ No If no, explain.

General

1. What did you like least about the course?

2. What did you like most about the course?

Additional Comments

Only students enrolled with the Alberta Distance Learning Centre need to complete the remaining questions.

1. Did you contact the Alberta Distance Learning Centre for help or information while doing your course?

   ☐ Yes ☐ No If yes, approximately how many times? ______

   Did you find the staff helpful?

   ☐ Yes ☐ No If no, explain.

   ____

   ____

2. Were you able to fax any of your assignment response pages?

   ☐ Yes ☐ No If yes, comment on the value of being able to do this.

   ____

   ____

3. If you mailed your assignment response pages, how long did it take for their return?

   ____

   ____

4. Was the feedback you received from your correspondence or distance learning teacher helpful?

   ☐ Yes ☐ No Please comment.

   ____

   ____

   ____

   ____

Thanks for taking the time to complete this questionnaire. Your feedback is important to us. Please return this questionnaire to the address on the right.

If you are enrolled at the Alberta Distance Learning Centre and will be mailing your Assignment Booklets to ADLC, you may return this questionnaire with the final Assignment Booklet in the course.

Learning Technologies Branch
Box 4000
Barrhead, Alberta
T7N 1P4

APPENDIX

Glossary
Suggested Answers
Image Credits

GLOSSARY

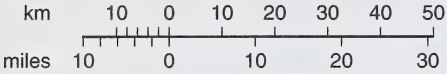
accuracy: an indication of how close a measurement comes to the true value

enlargement: a larger reproduction of an object

error of measurement: the difference between the actual size and the measured size of an object

internal feature: a feature of an object that is not fully visible from one of the views in an orthographic drawing

linear scale: a line with the scale values marked on it



orthographic drawing: a diagram that shows up to six views of an object

You can think of an orthographic drawing as the result of an object placed into a rectangular glass box and traced onto each side of the box. The resulting traces form the views of an orthographic diagram.

percentage error: the error expressed as a percentage of the given measurement

pictorial representation: a two-dimensional representation of a three-dimensional figure drawn from one perspective

rate: a certain quantity or amount of one thing considered in relation to one unit of another

ratio: a comparison of two or more quantities with the same units

reduction: a smaller reproduction of an object

scale: the ratio of the distance between two points on a map, model, or diagram to the distance between the actual locations

scale drawing: a drawing that either enlarges or reduces the original by a particular factor

scale factor: the ratio of corresponding sides of similar figures

significant digits: the meaningful digits of a number that represents a measurement

three-dimensional: having length, width, and depth

tolerance interval: the interval between the greatest and least acceptable measurements

tolerance range: the difference between the largest and the smallest acceptable measurements

two-dimensional: having length and width

uncertainty of measurement: one-half the precision of the instrument used to make the measurement

SUGGESTED ANSWERS

Activity 1: Two-Dimensional Enlargement and Reduction

1. Textbook exercises 1, 2, 4, and 5 of “Practice Your Prior Skills,” p. 358

1. To solve these problems, divide each number in the ratio by the greatest common factor (GCF).

a.

$$\begin{aligned} &76:100 \\ &76 \div 4 : 100 \div 4 \quad \longleftarrow \text{ The GCF of 76 and 100 is 4.} \\ &19:25 \end{aligned}$$

b.

$$\begin{aligned} &30:24 \\ &30 \div 6 : 24 \div 6 \quad \longleftarrow \text{ The GCF of 30 and 24 is 6.} \\ &5:4 \end{aligned}$$

c.

$$\begin{aligned} &1.5:0.5 \\ &1.5 \div 0.5 : 0.5 \div 0.5 \quad \longleftarrow \text{ The GCF of 1.5 and 0.5 is 0.5.} \\ &3:1 \end{aligned}$$

2. Write an equivalence of ratios for a 1650-kg elephant as $1:600 = x:1650$. Express this as an equation using fractions.

$$\begin{aligned} \frac{1}{600} &= \frac{x}{1650} \\ x &= \frac{1650}{600} \\ &= 2.75 \end{aligned}$$

The mass of the brain of a 1650-kg elephant is about 2.75 kg.

4. Recall that 1 km $= 100\ 000$ cm. Write an equivalence of ratios for a 3.5-km measure as $1:100\ 000 = 3.5 \text{ km} : x$. Express this as an equation using fractions.

$$\begin{aligned} \frac{1}{100\ 000} &= \frac{3.5}{x} \\ x &= 3.5(100\ 000) \\ &= 350\ 000 \text{ cm} \end{aligned}$$

Activity 1 (continued)

5. Recall that $1\ \text{m} = 1000\ \text{mm}$. You can write an equivalence of ratios for a 13.4-mm bug as $1:1000 = x:13.4$.

$$\begin{aligned}\frac{1}{1000} &= \frac{x}{13.4}\\ x &= \frac{13.4}{1000}\\ &= 0.0134\end{aligned}$$

A 13.4-mm beetle is 0.0134 m long.

2. Textbook exercises 1 to 7 of "Investigation 1: Using a Map Scale," p. 360

1. You have to change the measurements in metres to centimetres. Because $1\ \text{m} = 100\ \text{cm}$,

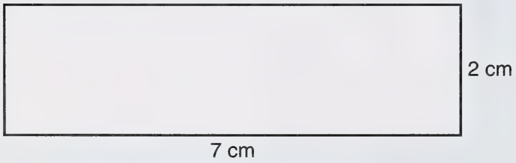
$$175\ \text{m} = 17\,500\ \text{cm}$$
$$50\ \text{m} = 5000\ \text{cm}$$

The dimensions of the building site is 17 500 cm by 5000 cm.

2. The scale factor is determined using the given ratio (1:2500); in this case, the scale factor is $\frac{1}{2500}$. Notice that the first number in the ratio becomes the numerator of the scale and the second number becomes the denominator.

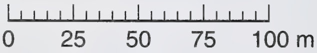
3. $17\,500\ \text{cm} \times \frac{1}{2500} = 7\ \text{cm}$ $\qquad$ $5000\ \text{cm} \times \frac{1}{2500} = 2\ \text{cm}$

In the scale drawing, the rectangle is 7 cm by 2 cm.

4. 

5. statement form: $1\ \text{cm} = 25\ \text{m}$

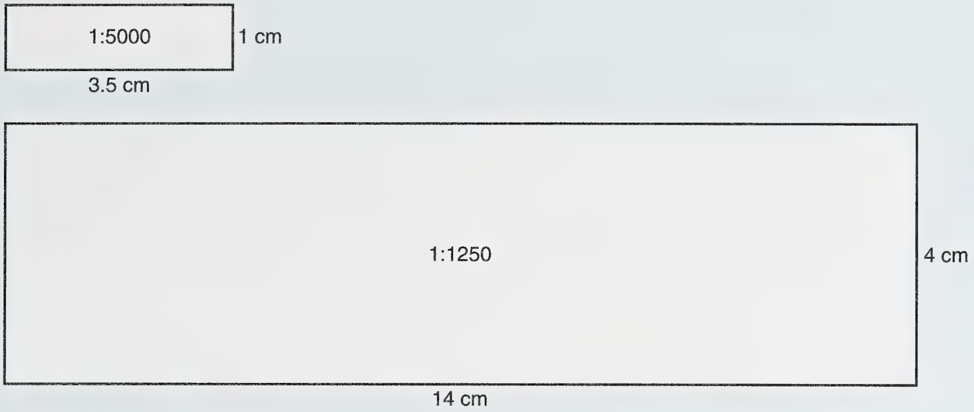
ratio form: $1:2500$

linear scale: 

6. **a.** If the scale is changed to $1:5000$, the dimensions of the diagram are halved.

b. If the scale is changed to $1:1250$, the dimensions of the diagram are doubled.

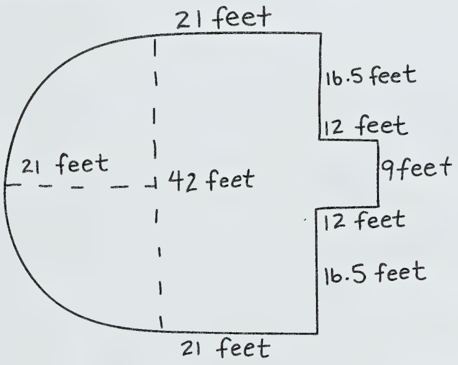
7.



3. Textbook exercises 1 to 7 of "Investigation 2: Creating a Floor Plan," p. 361

Answers will vary. Sample answers are given.

1. and 2.



3. $\frac{1}{4}$ inch $= 3$ feet

4. 3 feet $= 3$ feet $\times 12$ inches/foot

$= 36$ inches

$\therefore \frac{1}{4}:36 = 1:144$

The scale factor is $\frac{1}{144}$.

Activity 1 (continued)

5. $42\ \text{feet} = 42\ \text{feet} \times 12\ \text{inches/foot}$
$= 504\ \text{inches}$

$\therefore$ Scale dimension $= 504\ \text{inches} \times \frac{1}{144}$
$= 3.5\ \text{inches}$

$21\ \text{feet} = 21\ \text{feet} \times 12\ \text{inches/foot}$
$= 252\ \text{inches}$

$\therefore$ Scale dimension $= 252\ \text{inches} \times \frac{1}{144}$
$= 1.75\ \text{inches}$

$16.5\ \text{feet} = 16.5\ \text{feet} \times 12\ \text{inches/foot}$
$= 198\ \text{inches}$

$\therefore$ Scale dimension $= 198\ \text{inches} \times \frac{1}{144}$
$= 1.375\ \text{inches}$

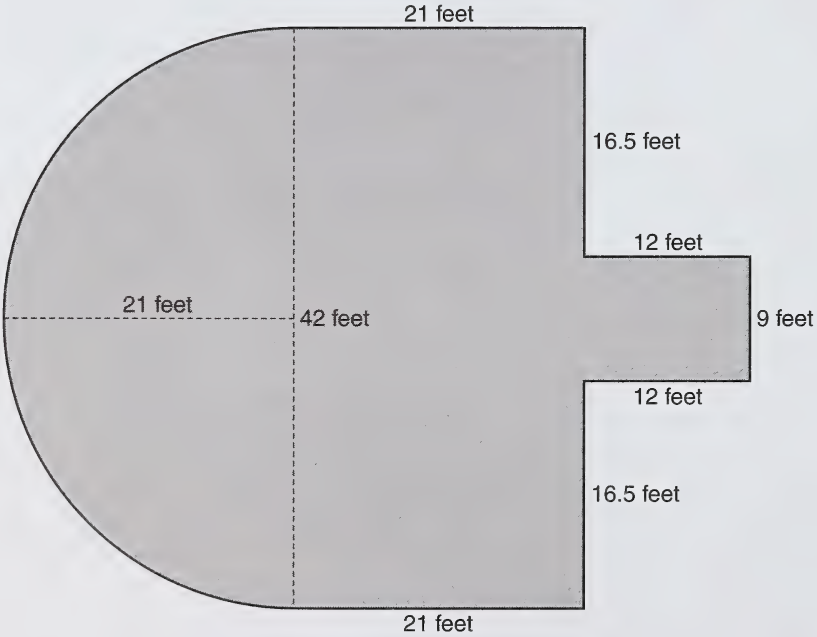
$12\ \text{feet} = 12\ \text{feet} \times 12\ \text{inches/foot}$
$= 144\ \text{inches}$

$\therefore$ Scale dimension $= 144\ \text{inches} \times \frac{1}{144}$
$= 1.0\ \text{inch}$

$9\ \text{feet} = 9\ \text{feet} \times 12\ \text{inches/foot}$
$= 108\ \text{inches}$

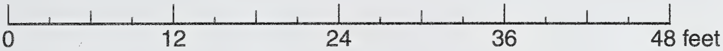
$\therefore$ Scale dimension $= 108\ \text{inches} \times \frac{1}{144}$
$= 0.75\ \text{inches}$

6.



7. **statement form:** $\frac{1}{4}$ inch $= 3$ feet **rate form:** $\dfrac{\frac{1}{4}\text{ inch}}{3\text{ feet}}$ **ratio form:** $1:144$

linear scale:



4. **Textbook exercises 1 to 6 of "Discussing the Ideas," p. 362**

Answers will vary. Sample answers are given.

1. There are several possibilities for the difference in the measurements. There may be a measurement error on the map reading, or the car odometer may have some error. Also, the 290 km was the straight-line distance on the map. The actual road probably has curves, hills, and valleys that add additional distance.

2. **statement form:** floor plans
rate form: solving proportions
ratio form: enlargements of tiny objects
linear scale: maps

3. The length and width in the drawing was determined by multiplying the length and width by the scale factor.

4. One method is to divide the width by 8.5 and choose a scale that is slightly smaller. (A standard page of paper is 8.5 inches wide.) You might have to modify this if your room is very long compared to its width. In this case, you could use your length divided by 11. (A standard page of paper is 11 inches long.)

5. If you were looking at reality being an enlargement of the map, the scale would reverse to $50\,000:1$.

6. In ratio form, the first number represents the new object and the second number reality. If the first number is smaller than the second, it is a reduction. (The new object is smaller than in reality.) If the first number is larger than the second, it is an enlargement. (The new object is bigger than in reality.)

5. **Textbook exercises 1.a., 2, 3, 5, 7, and 9 of "Exercises, Checking Your Skills," pp. 363 to 365**

1. **a.** 9 m at $1:50$ is 8 m at $1:50$ is

$$9\times\frac{1}{50}=0.18 \text{ m or } 18 \text{ cm} \qquad 8\times\frac{1}{50}=0.16 \text{ m or } 16 \text{ cm}$$

The scale drawing of the classroom should be 18 cm by 16 cm.

2. $\dfrac{18 \text{ m}}{2 \text{ m/cm}}=9 \text{ cm}$ $\dfrac{9 \text{ m}}{2 \text{ m/cm}}=4.5 \text{ cm}$

The scale drawing of the volleyball court is 9 cm by 4.5 cm.

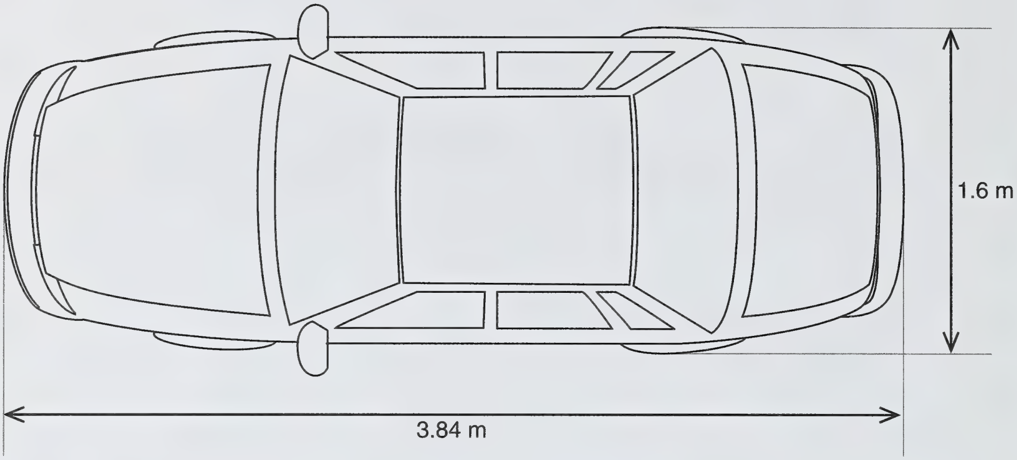
Activity 1 (continued)

3. **a.** $\text{length} = 3.84 \times \frac{1}{32}$

$= 0.12 \text{ m or } 12 \text{ cm}$

$\text{width} = 1.6 \times \frac{1}{32}$

$= 0.05 \text{ m or } 5 \text{ cm}$



b. $2.88 \text{ m} \times \frac{1}{32} = 0.09 \text{ m or } 9 \text{ cm}$

The wheels should be 9 cm apart.

5. Answers will vary. A sample answer is given for a paper clip with dimensions 2.8 cm by 0.7 cm.

a. The drawing will be an enlargement.

b. $\text{length} = 2.8 \text{ cm} \times 5$

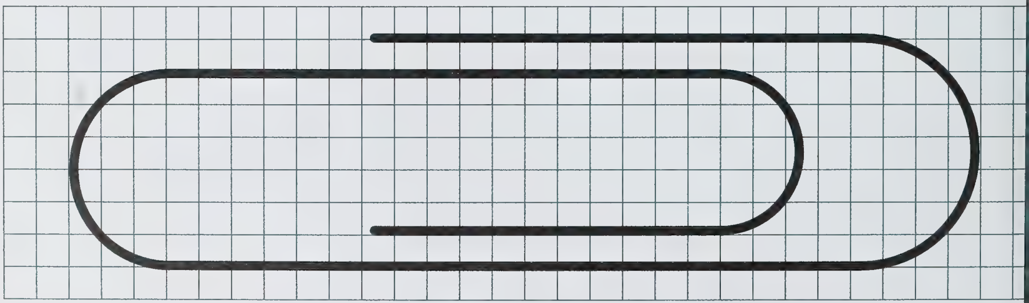
$= 14 \text{ cm}$

$\text{width} = 0.7 \text{ cm} \times 5$

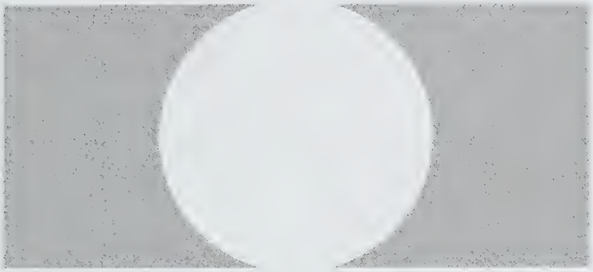
$= 3.5 \text{ cm}$

The dimensions of the scale drawing are 14 cm by 3.5 cm.

c.



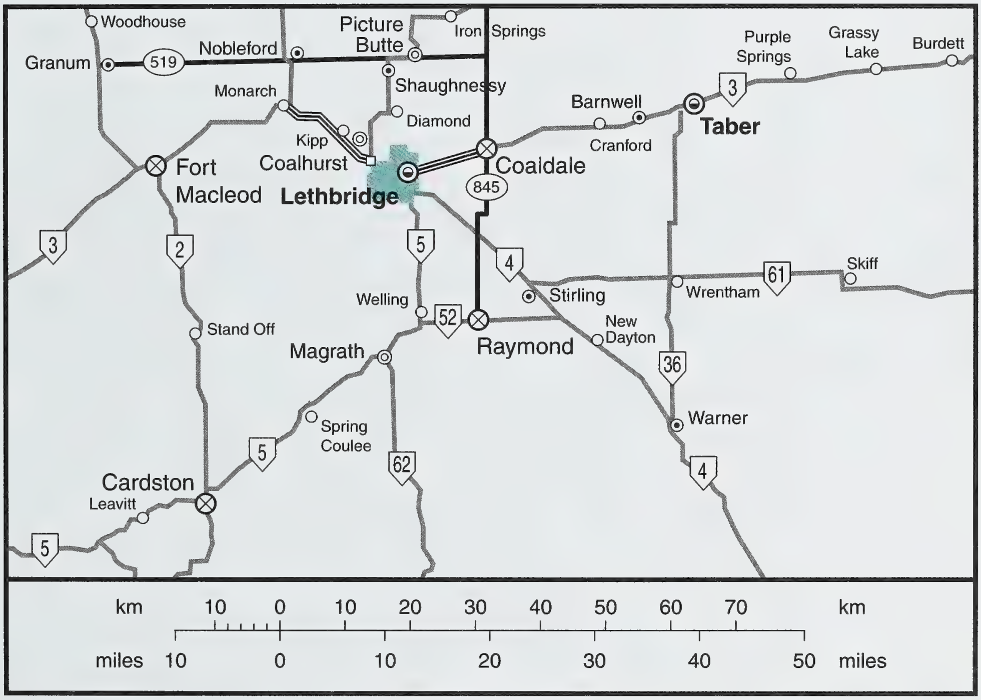
7.



The final yellow bar should be 9 cm wide and 4.05 cm high. The circle is 4.2 cm in diameter.

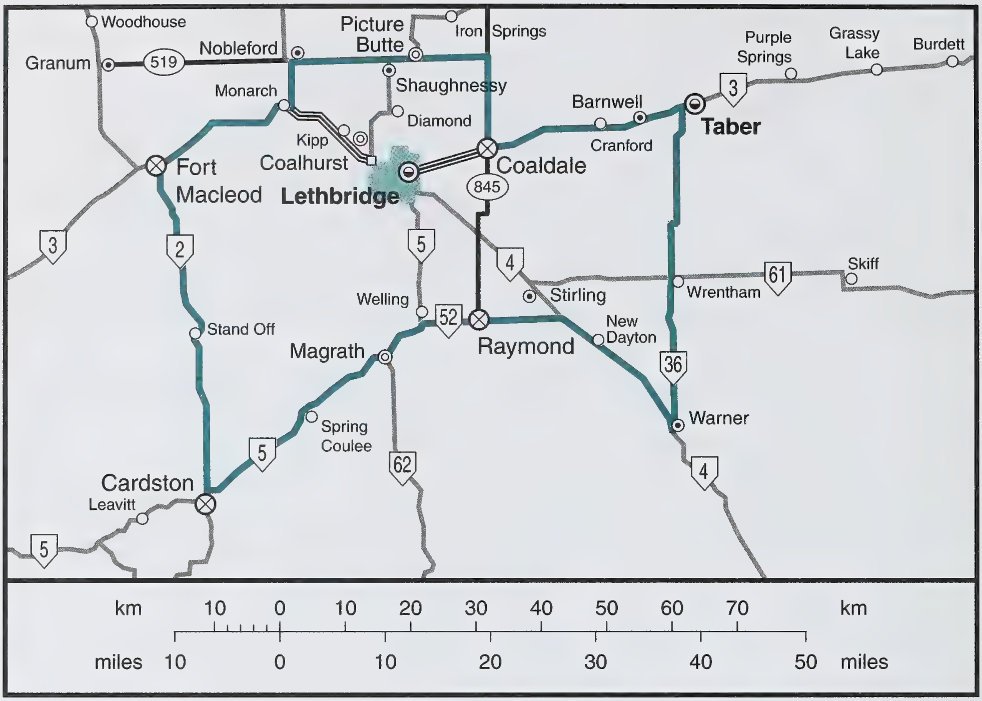
9. Answers will vary. Sample answers are given.

a. From Taber, plan a trip to the towns of Warner, Raymond, Cardston, Fort Macleod, Picture Butte, and then back to Taber.



Activity 1 (continued)

b. Here is a tracing of the route used.



c. Taber to Warner: 5.3 cm
Warner to Raymond: 4.4 cm
Raymond to Cardston: 5.3 cm
Cardston to Fort Macleod: 5.5 cm
Fort Macleod to Picture Butte: 5.0 cm
Picture Butte to Taber: 5.8 cm

TOTAL 31.3 cm

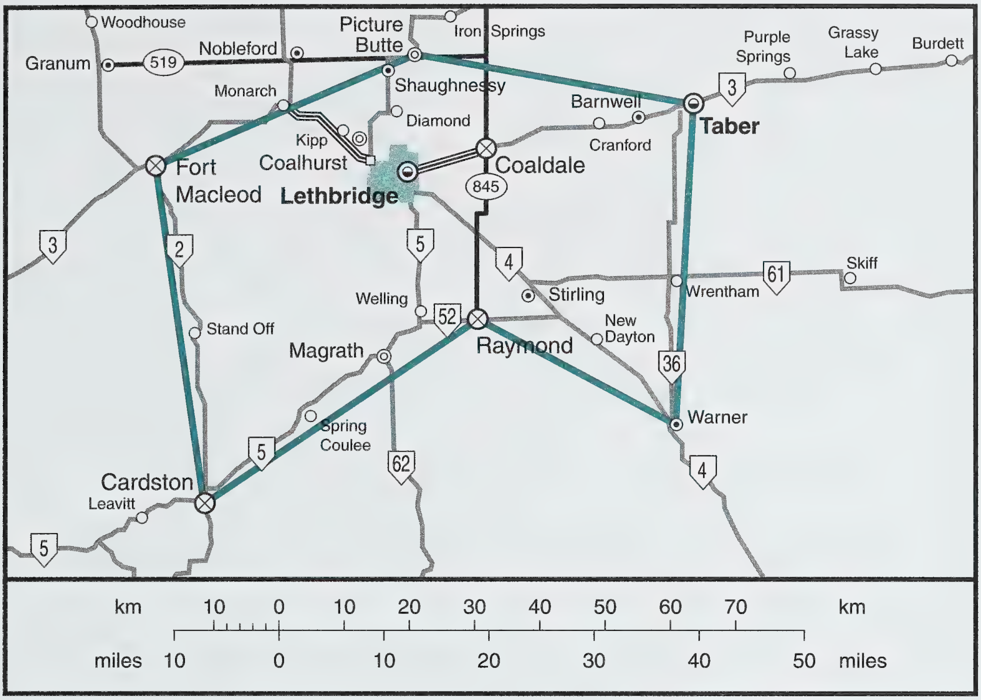
Since 7 cm represents 70 km on the road, the trip will have a distance of

$$\frac{7\text{ cm}}{70\text{ km}} = \frac{31.3\text{ cm}}{x}$$

$$\begin{aligned} x &= 31.3\text{ cm} \times \frac{70\text{ km}}{7\text{ cm}} \\ &= 313\text{ km} \end{aligned}$$

Because of the uncertainty of the measurements, the total distance is about 310 km.

d.



Distances as the crow flies measured on the map are

Taber to Warner:	5.0 cm
Warner to Raymond:	3.4 cm
Raymond to Cardston:	5.1 cm
Cardston to Fort Macleod:	5.3 cm
Fort Macleod to Picture Butte:	4.3 cm
Picture Butte to Taber:	4.3 cm
TOTAL	27.4 cm

The actual distance is calculated as $27.4 \text{ cm} \times \frac{70 \text{ km}}{7 \text{ cm}} = 274 \text{ km}$. This should be reported as 270 km. "Distance as the crow flies" is generally shorter.

6. Textbook exercise "Communicating the Ideas," p. 365

Answers will vary. Some examples of the use of scale drawings are as follows:

- house plans for a builder: scale $\doteq \frac{1}{50}$
- road maps: scale factor $\doteq \frac{1}{1\,000\,000}$
- plans for planes to be used by model makers: scale factor $\doteq \frac{1}{24}$

Activity 2: Portraying Three-Dimensional Objects

1. **Textbook exercises 1 to 3 of "Discussing the Ideas," p. 369**

    1. **Advantages**

        - A pictorial diagram helps visualize an object in three dimensions.
        - An orthographic diagram shows all views accurately.

        Disadvantages

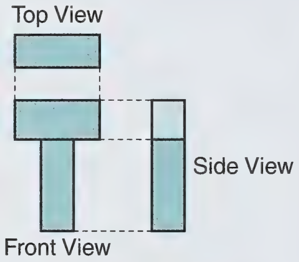
        - A pictorial diagram may show features in a distorted way and may not show hidden features.
        - An orthographic diagram is not always easy to understand in a three-dimensional sense.

    2. Drawings of objects that have a circular cross-section will only require two points of view. Complex objects, like a car, will need more views to show all of the features.

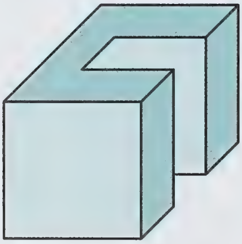
    3. Example 3 shows that you need to first find the actual sizes from the scale diagram. Then you need to change these values using the new scale factor. The final step involves using the ratio of the corresponding lengths to find a new scale for the diagram.

2. **Textbook exercises 1.a., 2.a., 3, 4, and 7 of "Exercises: Checking Your Skills," pp. 370 and 371**

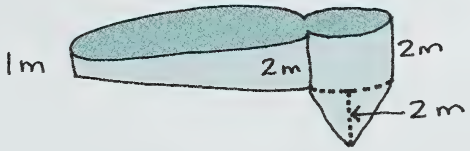
    1. **a.**



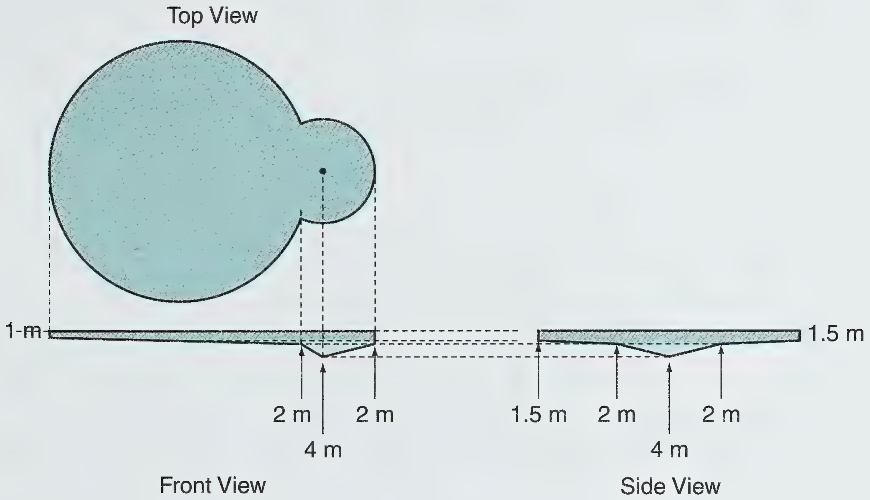
    2. **a.**



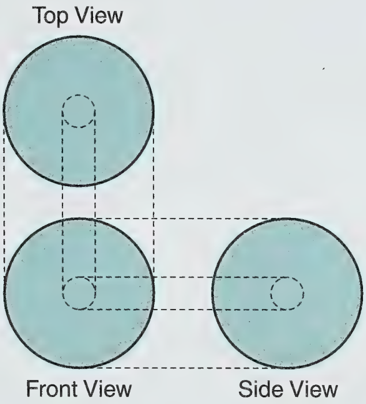
3. **a.** Here is a rough sketch of the pool.



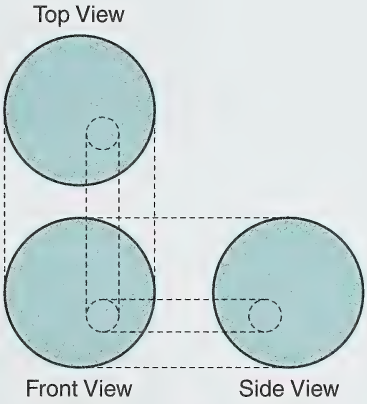
b. The orthographic drawing follows. The scale is 1:1000.



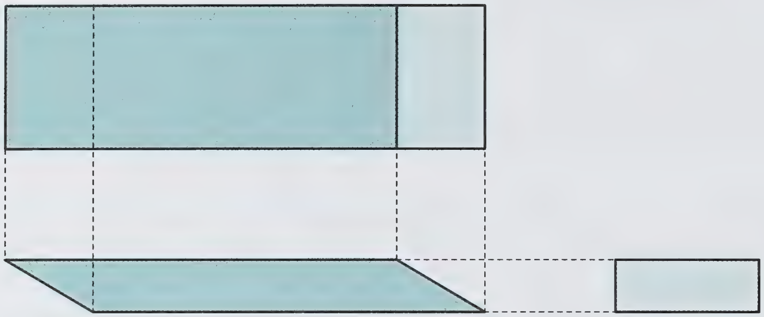
4. **a.**



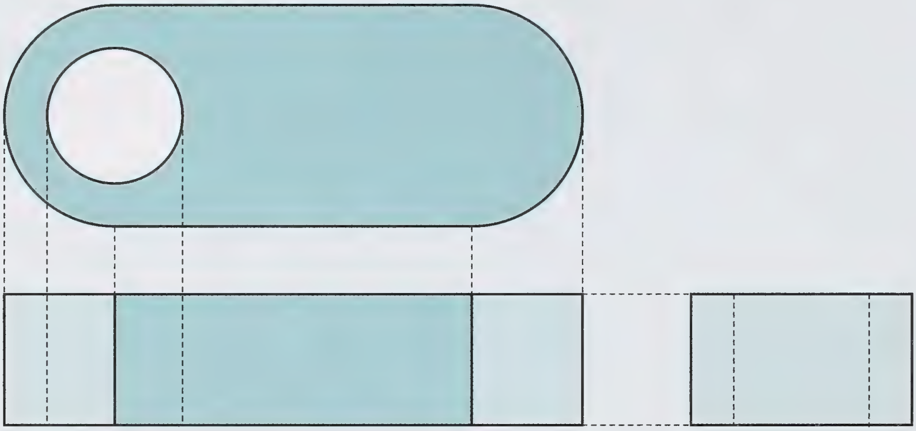
b.

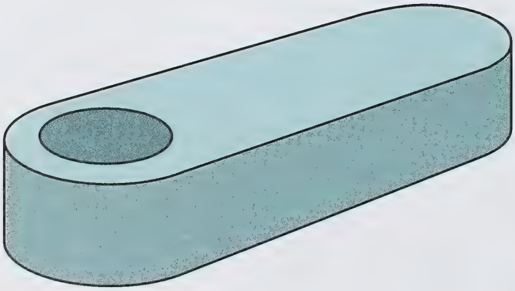


7. Orthographic diagrams will vary. A sample answer using an eraser is given.



3. Textbook exercise "Communicating the Ideas," p. 371





The orthographic views show that the hole goes through the object and shows how the hole is placed in the object. The pictorial diagram shows the circular hole as an ellipse and doesn't show that the hole goes all the way through the object. Both views show the curved ends of the object.

Activity 3: Accuracy in Measurement

1. Uncertainty of measurement is an approximation of the measurement error.

2. The precision of a measuring instrument is the smallest scale division on it.

3. The zero is added to provide the same number of places to the right of the decimal in the measurement as there are in the uncertainty of the measurement. The zero also shows that the digit to the left of it is known with certainty.

4. **Textbook exercises 1 to 10 of "Investigation 1: Adding and Subtracting Measurements with the Same Precision," p. 374**

    1. The width is 19.5 cm, and the length is 24.8 cm.

    2. The width is (19.50 ± 0.05) cm, and the length is (24.80 ± 0.05) cm.

    3. The width is between 19.45 cm and 19.55 cm. The length is between 24.75 cm and 24.85 cm.

    4. $19.45 + 24.75 = 44.20 \text{ cm}$ $\quad$ $19.55 + 24.85 = 44.40 \text{ cm}$

        The semi-perimeter lies between 44.20 cm and 44.40 cm.

    5. $\frac{44.20 + 44.40}{2} = 44.30 \text{ cm}$

        The average length of the semi-perimeter is 44.30 cm.

    6. $44.30 - 44.20 = 0.10 \text{ cm}$

        The difference between the average and minimum lengths for the semi-perimeter is 0.10 cm. The semi-perimeter can be written as (44.30 ± 0.10) cm.

    7. $24.75 - 19.55 = 5.20 \text{ cm}$ $\quad$ $24.85 - 19.45 = 5.40 \text{ cm}$

        The limits of the difference of length and width of the textbook are 5.20 cm and 5.40 cm. Using ± notation, this becomes (5.30 ± 0.10) cm.

    8. The uncertainties of the sum of the length and width is equal to the sum of the uncertainties of the original measurements. In this case, 0.10 cm (which is equal to $0.05 \text{ cm} + 0.05 \text{ cm}$).

    9. The uncertainties of the difference of the length and width is equal to the sum of the uncertainties of the original measurements. In this case, 0.10 cm (which is equal to $0.05 \text{ cm} + 0.05 \text{ cm}$).

    10. The uncertainty of a sum or difference is the sum of the uncertainties of the original measurements.

5. **Textbook exercises 1 to 4 of "Investigation 2: Adding and Subtracting Measurements with Different Precision," p. 375**

   1. Repeat exercises 1 to 10 of Investigation 1 with different measuring devices.

      1. The width is 19.5 cm, and the length is 25 cm.

      2. The width is 19.50 ± 0.05 cm, and the length is 25.0 ± 0.5 cm.

      3. The width is between 19.45 cm and 19.55 cm. The length is between 24.5 cm and 25.5 cm.

      4. $19.45 + 24.5 = 43.95$ cm $\qquad$ $19.55 + 25.5 = 45.05$ cm

         The semi-perimeter lies between 43.95 cm and 45.05 cm.

      5. $\dfrac{43.95 + 45.05}{2} = 44.5$ cm

         The average length of the semi-perimeter is 44.5 cm.

      6. $44.5 - 43.95 = 0.55$ cm

         The difference between the average and minimum lengths for the semi-perimeter is 0.55 cm. The semi-perimeter should not be written as 44.50 ± 0.55 cm, since this is more precise than the least-precise measurement. In this case, you should report the semi-perimeter as 44.5 ± 0.6 cm. **Note:** The uncertainty here has been rounded to 0.6 cm. The textbook dropped the 5 in 0.55.

      7. $24.5 - 19.55 = 4.95$ cm $\qquad$ $25.5 - 19.45 = 6.05$ cm

         The limits of the difference of length and width of the textbook are 4.95 cm and 6.05 cm. Using ± notation, this becomes 5.5 ± 0.6 cm, again using the least precise of the measurements.

      8. The uncertainties of the sum of the length and width is equal to the sum of the uncertainties of the original measurements. In this case, start with 0.55 cm (which is equal to $0.05 \text{ cm} + 0.5 \text{ cm}$); then round up to match the precision of the least-precise uncertainty, giving 0.6 cm.

      9. The uncertainties of the difference of length and width is equal to the sum of the uncertainties of the original measurements. In this case, start with 0.55 cm (which is equal to $0.05 \text{ cm} + 0.5 \text{ cm}$); then round up to 0.6 cm to match the precision of the least-precise uncertainty.

      10. The uncertainty of a sum or difference is the sum of the uncertainties of the original measurements.

2. The conclusions are the same.

3. When you add or subtract two measurements, the uncertainties are added.

4. A measurement of (0.3070 ± 0.0005) m means the value is between 0.3065 m and 0.3075 m. A measurement of (4.120 ± 0.005) m means the value is between 4.115 m and 4.125 m.

   $0.3065 + 4.115 = 4.4215$ m $\qquad$ $0.3075 + 4.125 = 4.4325$ m

   The sum is between 4.4215 m and 4.4325 m. The nominal value is 4.427 ± 0.006 m.

   $4.115 - 0.3075 = 3.8075$ m $\qquad$ $4.125 - 0.3065 = 3.8185$ m

   The difference is between 3.8075 m and 3.8185 m. The nominal value is 3.813 ± 0.006 m.

   The conclusion in question three seems correct.

   Note: The sum and difference given here agree with the answer given in the textbook on page 458. The uncertainty here, however, has been rounded to three decimal places (not to four decimal places as in the textbook). It is best to maintain only one digit in the uncertainty.

6. Textbook exercises 1 to 5 of "Discussing the Ideas," p. 377

1. While the tool used to make the measurements contributes to the uncertainty of the final result, other factors affect it as well, such as your ability to use the measuring tool and your accuracy in using the tool.

2. The uncertainty is taken as half of the precision of the ruler. If the ruler is marked in millimetres, the uncertainty is half a millimetre (0.5 mm).

3. The uncertainty is the same whether you add or subtract measurements. The range of possible values increases when you operate on measurements.

4. The precision of a measurement cannot be increased after it is taken. So, the least precise of your measurements is the most you can say about the precision of a sum or difference.

5. Measurement error is the difference between a measurement and an actual value. Since you cannot determine the actual value, you cannot calculate the measurement error. Uncertainty of a measurement is used as an approximation to the measurement error.

Activity 3 (continued)

7. a. Textbook exercises 1, 2, 3, 5, and 6 of "Exercises: Checking Your Skills," pp. 378 and 379

1.

	Measurement Tool	Measurement	Uncertainty
a.	metric ruler	24.7 cm	0.05 cm
b.	graduated cylinder	81 mL	0.5 mL
c.	micrometer	9.23 mm	0.005 mm
d.	yardstick	13.2 inches	0.05 inches
e.	Imperial scale	2.4 pounds	0.05 pounds
f.	thermometer	101.8°C	0.05°C

Note: For 1.a., the metric ruler is assumed to be marked in millimetres.

2. First, carry out the required operations; then round to the appropriate number of decimal places.

a.

$$\begin{aligned}(26.230 \pm 0.005)\ \text{mm} + (11.170 \pm 0.005)\ \text{mm} &= (26.230 + 11.170) \pm (0.005 + 0.005)\ \text{mm}\\ &= (37.400 \pm 0.010)\ \text{mm}\end{aligned}$$

$$\begin{aligned}(26.230 \pm 0.005)\ \text{mm} - (11.170 \pm 0.005)\ \text{mm} &= (26.230 - 11.170) \pm (0.005 + 0.005)\ \text{mm}\\ &= (15.060 \pm 0.010)\ \text{mm}\end{aligned}$$

b.

$$\begin{aligned}(517.0 \pm 0.5)\ \text{m} + (403.0 \pm 0.5)\ \text{m} &= (517.0 + 403.0) \pm (0.5 + 0.5)\ \text{m}\\ &= (920.0 \pm 1.0)\ \text{m}\end{aligned}$$

$$\begin{aligned}(517.0 \pm 0.5)\ \text{m} - (403.0 \pm 0.5)\ \text{m} &= (517.0 - 403.0) \pm (0.5 + 0.5)\ \text{m}\\ &= (114.0 \pm 1.0)\ \text{m}\end{aligned}$$

c.

$$\begin{aligned}(11.0 \pm 0.5)\ \text{km/h} + (11.0 \pm 0.5)\ \text{km/h} &= (11.0 + 11.0) \pm (0.5 + 0.5)\ \text{km/h}\\ &= (22.0 \pm 1.0)\ \text{km/h}\end{aligned}$$

$$\begin{aligned}(11.0 \pm 0.5)\ \text{km/h} - (11.0 \pm 0.5)\ \text{km/h} &= (11.0 - 11.0) \pm (0.5 + 0.5)\ \text{km/h}\\ &= (0.0 \pm 1.0)\ \text{km/h}\end{aligned}$$

d.
$$\begin{aligned}(5.30 \pm 0.05)\ \text{g} + (3.240 \pm 0.005)\ \text{g} &= (5.30 + 3.240) \pm (0.05 + 0.005)\ \text{g}\\ &= (8.540 \pm 0.055)\ \text{g}\\ &= (8.54 \pm 0.06)\ \text{g} \quad \longleftarrow \text{precision to the least-precise measurement}\end{aligned}$$

$$\begin{aligned}(5.30 \pm 0.05)\ \text{g} - (3.240 \pm 0.005)\ \text{g} &= (5.30 - 3.240) \pm (0.05 + 0.005)\ \text{g}\\ &= (2.060 \pm 0.055)\ \text{g}\\ &= (2.06 \pm 0.06)\ \text{g} \quad \longleftarrow \text{precision to the least-precise measurement}\end{aligned}$$

e.
$$\begin{aligned}(7.2160 \pm 0.0005)\ \text{s} + (13.40 \pm 0.05)\ \text{s} &= (7.2160 + 13.40) \pm (0.0005 + 0.05)\ \text{s}\\ &= (20.6160 \pm 0.0505)\ \text{s}\\ &= (20.62 \pm 0.06)\ \text{s} \quad \longleftarrow \text{Always round the uncertainty up.}\end{aligned}$$

$$\begin{aligned}(7.2160 \pm 0.0005)\ \text{s} - (13.40 \pm 0.05)\ \text{s} &= (7.2160 - 13.40) \pm (0.0005 + 0.05)\ \text{s}\\ &= (-6.184 \pm 0.0505)\ \text{s}\\ &= (-6.18 \pm 0.06)\ \text{s} \quad \longleftarrow \text{Always round the uncertainty up.}\end{aligned}$$

f.
$$\begin{aligned}\$(280\ 000 \pm 20\ 000) + \$(115\ 000 \pm 2500) &= \$(280\ 000 + 115\ 000) \pm (20\ 000 + 2500)\\ &= \$(395\ 000 \pm 22\ 500)\\ &= \$(400\ 000 \pm 30\ 000) \quad \longleftarrow \text{Always round the uncertainty up.}\end{aligned}$$

$$\begin{aligned}\$(280\ 000 \pm 20\ 000) + \$(115\ 000 \pm 2500) &= \$(280\ 000 - 115\ 000) \pm (20\ 000 + 2500)\\ &= \$(165\ 000 \pm 22\ 500)\\ &= \$(170\ 000 \pm 30\ 000) \quad \longleftarrow \text{Always round the uncertainty up.}\end{aligned}$$

3. a. The precision of the measurements appears to be 1 cm; so, the uncertainty is 0.5 cm.

$$\begin{aligned}\therefore \text{Perimeter} &= (3 \pm 0.5)\ \text{cm} + (4 \pm 0.5)\ \text{cm} + (5 \pm 0.5)\ \text{cm}\\ &= (3 + 4 + 5) \pm (0.5 + 0.5 + 0.5)\ \text{cm}\\ &= (12 \pm 1.5)\ \text{cm}\end{aligned}$$

The perimeter will be between 10.5 cm and 13.5 cm.

Activity 3 (continued)

b. The precision of the measurements appears to be 1 mm; so, the uncertainty is 0.5 mm.

$$\begin{aligned}\therefore \text{ Perimeter} &= (12 \pm 0.5)\text{ mm} + (12 \pm 0.5)\text{ mm} + (12 \pm 0.5)\text{ mm} + (12 \pm 0.5)\text{ mm}\\ &= (12+12+12+12) \pm (0.5+0.5+0.5+0.5)\text{ mm}\\ &= (48 \pm 2.0)\text{ mm}\end{aligned}$$

The perimeter of the square lies between 46 mm and 50 mm.

c. The precision appears to be 1 m; so, the uncertainty is 0.5 m.

$$\begin{aligned}\therefore \text{ Perimeter} &= (12 \pm 0.5)\text{ m} + (12 \pm 0.5)\text{ m} + (8 \pm 0.5)\text{ m} + (8 \pm 0.5)\text{ m}\\ &= (12+12+8+8) \pm (0.5+0.5+0.5+0.5)\text{ m}\\ &= (40 \pm 2.0)\text{ m}\end{aligned}$$

The perimeter of the rectangle lies between 38 m and 42 m.

5. a. A cube has 12 edges; so, the model will require 12 lengths of 4 cm. The uncertainty of each measurement is 0.5 cm; so, the total amount of wire is (4 ± 0.5) cm added 12 times (or multiplied by 12). This gives a total of (48 ± 6.0) cm. Therefore, the minimum amount of wire needed is 42 cm.

b. The box has 4 lengths of wire measuring 5.2 cm, 4 lengths of wire measuring 3.1 cm, and 4 lengths of wire measuring 6.5 cm. The uncertainty of each measurement is 0.05 cm.

$$\begin{aligned}\therefore \text{ Amount of wire} &= (4 \times 5.2 + 4 \times 3.1 + 4 \times 6.5) \pm (12 \times 0.05)\\ &= (59.2 \pm 0.60)\text{ cm}\end{aligned}$$

Therefore, the minimum amount of wire needed is 58.6 cm.

c. The square pyramid has 4 lengths of wire measuring 3 inches and 4 lengths of wire measuring 5 inches. The uncertainty of each measurement is 0.5 inches.

$$\begin{aligned}\therefore \text{ Amount of wire} &= (4 \times 3 + 4 \times 5) \pm (8 \times 0.5)\\ &= (32 \pm 4.0)\text{ inches}\end{aligned}$$

Therefore, the minimum amount of wire needed is 28 inches.

6. **a.** The uncertainty for each measurement is 0.05 cm.

   b. $$\begin{aligned}\text{Total length} &= (11.3 \pm 0.05) + (5.9 \pm 0.05) - (0.50 \pm 0.05)\\ &= (11.3 + 5.9 - 0.50) \pm (3 \times 0.05)\\ &= (16.7 \pm 0.15)\ \text{cm}\end{aligned}$$

   The total length of the tubing is between 16.55 cm and 16.85 cm. The uncertainty of the total length is ±0.15 cm.

b. Textbook exercise 8 of "Exercises: Extending Your Thinking," p. 379

8. Because 3000 pieces of track are laid, there will be a total of 2999 spaces.

$$\begin{aligned}\therefore\ \text{Total length} &= (\text{Total length of track} \pm \text{Uncertainty}) + (\text{Total amount of space} \pm \text{Uncertainty})\\ &= (3000 \times 12.00 \pm 3000 \times 0.05)\ \text{m} + (2999 \times 1.0 \pm 2999 \times 0.2)\ \text{cm}\\ &= (36\,000 \pm 150.00)\ \text{m} + (2999 \pm 599.8)\ \text{cm}\\ &= (36\,000 \pm 150.00)\ \text{m} + (29.99 \pm 5.998)\ \text{m}\\ &= (36\,029.99 \pm 155.998)\ \text{m}\\ &= (36\,029.99 \pm 156.00)\ \text{m}\end{aligned}$$

The length of the track will be between 35 873.99 m and 36 185.99 m. Notice that the values are rounded to the precision of the least-precise measurement and that the uncertainty is rounded up and to the precision of the least-precise uncertainty.

8. Textbook exercise "Communicating the Ideas," p. 379

Standards of measurement are what allow a person to design an object, with many parts and several others building the parts, knowing that the parts will fit together correctly when it comes time for assembly. The lack of such a measuring system would lead to many problems of varying magnitude. If the piston manufacturer, for example, made the pistons too tall, the engine could self-destruct. If the manufacturers of the cable for an elevator made the cables too small, the elevator could fall. The modern metric and imperial measuring systems, along with knowledge of the error of measurements, allows such occurrences to be very rare.

Activity 4: Using Percentage to Describe Error

1. Textbook exercises 1 to 4 of "Practise Your Prior Skills," p. 382

1. **a.** Diagram B has the shots grouped closely together (good precision) but away from the bull's eye (poor accuracy).

   b. Diagram C has the shots grouped closely together (good precision) and close to the bull's eye (good accuracy).

Activity 4 (continued)

2. $2\frac{7}{8} = 2 + \frac{7}{8}$ or $2\frac{7}{8} = \frac{23}{8}$

$$\begin{aligned} 2\tfrac{7}{8} &= 2 + \tfrac{7}{8} \\ &= 2 + 0.875 \\ &= 2.875 \end{aligned} \qquad \text{or} \qquad \begin{aligned} 2\tfrac{7}{8} &= \tfrac{23}{8} \\ &= 2.875 \end{aligned}$$

3. $\frac{3}{6000} \times 100\% = 0.05\%$

4. The discount is $\$195 - \$156 = \$39$.

$$\therefore \frac{39}{195} \times 100\% = 20\%$$

The discount is 20% of the original price.

2. Answers will vary. Sample factors are given.

- the relative costs of the machines
- the cost per non-defective part
- the training needed to use the machines
- the speed of the machines
- changes to the factory required by each machine
- delivery time for the machines
- outsourcing (Is the machine even needed?)

3. **Textbook exercises 1 to 4 of "Discussing the Ideas," p. 385**

1. Compare the accuracy of these measurements by using percentage error.

   To determine percentage error, divide the uncertainty by the measurement and multiply the result by 100. Do this for each uncertainty and measurement. Compare the resulting percentage errors to determine the accuracy of the measurements.

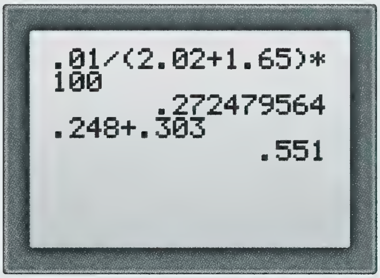
2. Percentage error has no units because it is a percentage calculated from the ratio of two measurements with the same units.

3. The uncertainty of the measurements would be the same because the same instrument was used in making the measurements. The percentage error would be smaller for the larger measurement because you are dividing by a larger number.

4. The uncertainty of a sum when given as a percent does not equal the sum of the percentage errors. Suppose the semi-perimeter of the table top in question 3 is considered. (Recall that semi-perimeter is the sum of the length and width.)

The uncertainty of the semi-perimeter is $0.005\ \text{m} + 0.005\ \text{m} = 0.010\ \text{m}$.

$$\begin{aligned}\therefore \text{ Percentage error} &= 0.010 \div (2.02 + 1.65) \times 100\% \\ &= 0.2725\%\end{aligned}$$



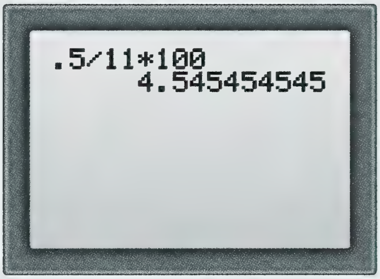
```
.01/(2.02+1.65)*
100
        .272479564
.248+.303
              .551
```

This is not the sum of 0.248% and 0.303%. Actually, since the sum of the measurements gives a larger divisor, the percentage error for the sum of the uncertainties is less than the percentage error for the individual uncertainties.

4. a. Textbook exercises 1, 2, 3, 5, and 6 of "Exercises: Checking Your Skills," pp. 385 and 386

1. a. The precision is 1 cm; so, the uncertainty is 0.5 cm.

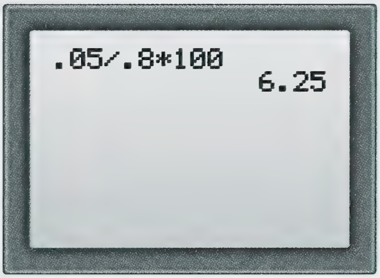
$$\begin{aligned}\therefore \text{ Percentage error} &= \frac{0.5}{11} \times 100\% \\ &\doteq 4.5\%\end{aligned}$$



```
.5/11*100
       4.545454545
```

b. The precision is 0.1 g; so, the uncertainty is 0.05 g.

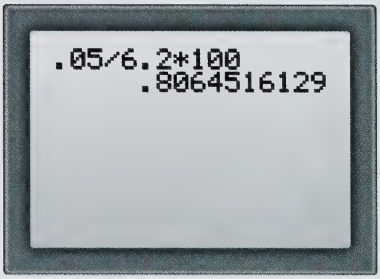
$$\begin{aligned}\therefore \text{ Percentage error} &= \frac{0.05}{0.8} \times 100\% \\ &= 6.25\%\end{aligned}$$



```
.05/.8*100
              6.25
```

c. The precision is 0.1 kg; so, the uncertainty is 0.05 kg.

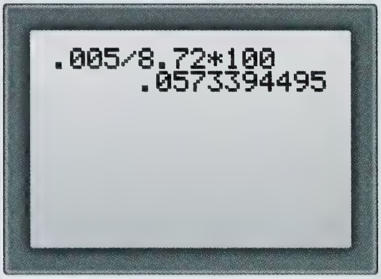
$$\begin{aligned}\therefore \text{ Percentage error} &= \frac{0.05}{6.2} \times 100\% \\ &\doteq 0.81\%\end{aligned}$$



```
.05/6.2*100
           .8064516129
```

d. The precision is 0.01 m; so, the uncertainty is 0.005 m.

$$\begin{aligned}\therefore \text{ Percentage error} &= \frac{0.005}{8.72} \times 100\% \\ &\doteq 0.057\%\end{aligned}$$



```
.005/8.72*100
           .0573394495
```

2. **a.** The uncertainty is 0.05 cm.

$$\begin{aligned}\therefore \text{ Percentage error} &= \frac{0.05}{24.7} \times 100\% \\ &\doteq 0.20\%\end{aligned}$$

b. The uncertainty is 0.5 mL.

$$\begin{aligned}\therefore \text{ Percentage error} &= \frac{0.5}{81} \times 100\% \\ &\doteq 0.6\%\end{aligned}$$

c. The uncertainty is 0.005 mm.

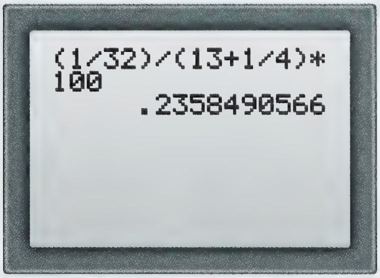
$$\begin{aligned}\therefore \text{ Percentage error} &= \frac{0.005}{9.23} \times 100\% \\ &\doteq 0.054\%\end{aligned}$$

d. The uncertainty is 0.05°C.

$$\begin{aligned}\therefore \text{ Percentage error} &= \frac{0.05}{101.8} \times 100\% \\ &\doteq 0.05\%\end{aligned}$$

e. The uncertainty is $\frac{1}{32}$ inch.

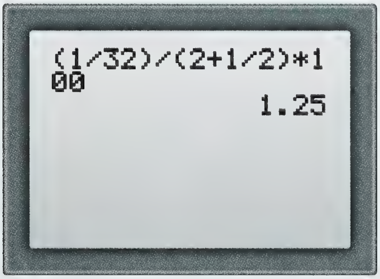
$$\therefore \text{ Percentage error} = \frac{\frac{1}{32}}{13\frac{1}{4}} \times 100\% \doteq 0.24\%$$



```
(1/32)/(13+1/4)*
100
           .2358490566
```

f. The uncertainty is $\frac{1}{32}$ pound.

$$\therefore \text{ Percentage error} = \frac{\frac{1}{32}}{2\frac{1}{2}} \times 100\% = 1.25\%$$



```
(1/32)/(2+1/2)*1
00
                  1.25
```

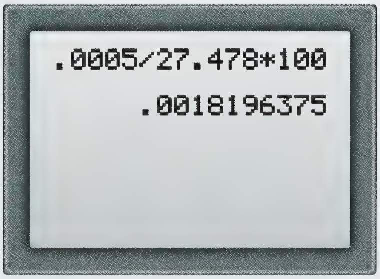
3. $$\text{Percentage error (mass)} = \frac{0.05}{32.4} \times 100\% \doteq 0.15\%$$

$$\text{Percentage error (height)} = \frac{0.005}{1.23} \times 100\% \doteq 0.407\%$$

The measurement of mass has the lower percentage error.

5. The uncertainty is 0.0005 g. Therefore, the greatest possible mass is 27.4785 g and the least possible mass is 27.4775 g.

$$\text{Percentage error} = \frac{0.0005}{27.478} \times 100\% \doteq 0.0018\%$$



```
.0005/27.478*100
           .0018196375
```

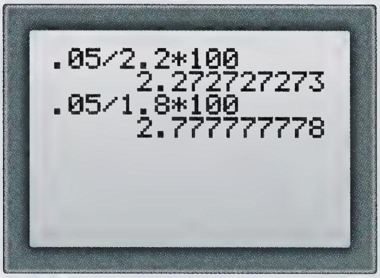
The mass of the container, using percentage error, is $27.478 \pm 0.0018\%$.

Activity 4 (continued)

6. The length of the stamp is (2.2 ± 0.05) cm, and the width of the stamp is (1.8 ± 0.05) cm.

$$\text{Percentage error (length)} = \frac{0.05}{2.2} \times 100\%$$
$$\doteq 2.27\%$$

$$\text{Percentage error (width)} = \frac{0.05}{1.8} \times 100\%$$
$$\doteq 2.78\%$$



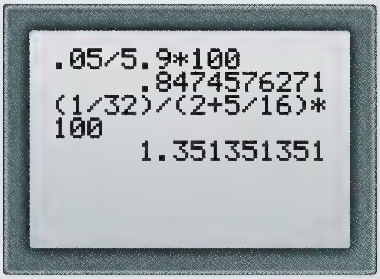
```
.05/2.2*100
        2.272727273
.05/1.8*100
        2.777777778
```

The length of the stamp is $2.2\ \text{cm} \pm 2.27\%$, and the width is $1.8\ \text{cm} \pm 2.78\%$.

b. Textbook exercise 8 of "Exercises: Entending Your Thinking," p. 386

8. $$\text{Percentage error (metric)} = \frac{0.05}{5.9} \times 100\%$$
$$\doteq 0.85\%$$

$$\text{Percentage error (imperial)} = \frac{\frac{1}{32}}{2\frac{5}{16}} \times 100\%$$
$$\doteq 1.35\%$$



```
.05/5.9*100
        .8474576271
(1/32)/(2+5/16)*
100
        1.351351351
```

The metric measurement has the smaller percentage error in this case.

5. Textbook exercise "Communicating the Ideas," p. 386

Percentage error is used to compare the uncertainty associated with a measurement because it is a common system that gives a simple expression of the error. It allows you to compare errors even when different measuring systems are used, and it is often easier to understand than other methods of stating error.

Activity 5: Describing Error in Products and Quotients

1. a. Textbook exercise 2 of "Discussing the Ideas," p. 390

2. The percentage error for a volume found from three measurements will be the sum of the percentage errors of each of the individual measurements.

b. Textbook exercises 1, 2, 3, and 5 of "Exercises: Checking Your Skills," pp. 391 and 392

1. a. $\text{Error in length} = 420 \times 0.015 = 6.3\ \text{cm}$ $\quad$ $\text{Error in width} = 170 \times 0.02 = 3.4\ \text{cm}$

The maximum area is the product of the maximum length and the maximum width.

$\text{Maximum length} = 420 + 6.3 = 426.3\ \text{cm}$ $\quad$ $\text{Maximum width} = 170 + 3.4 = 173.4\ \text{cm}$

$$\therefore \text{Maximum area} = 426.3 \times 173.4 = 73\,920.42\ \text{cm}^2$$

The minimum area is the product of the minimum length and the minimum width.

$\text{Minimum length} = 420 - 6.3 = 413.7\ \text{cm}$ $\quad$ $\text{Minimum width} = 170 - 3.4 = 166.6\ \text{cm}$

$$\therefore \text{Minimum area} = 413.7 \times 166.6 = 68\,922.42\ \text{cm}^2$$

b. The percentage error for the area of the table is approximately $1.5\% + 2\% = 3.5\%$.

You could also calculate it as

$$\frac{\frac{73\,920.42 - 68\,922.42}{2}}{(420 \times 170)} \times 100\% = 3.5\%$$

2. a. The uncertainty is 25 mL (half of 50 mL).

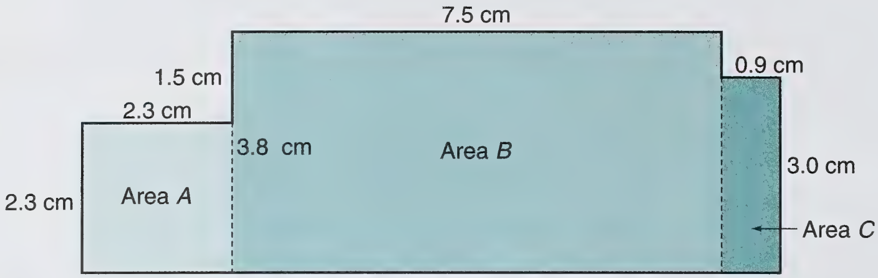
b. $$\text{Percentage error} = \frac{25\ \text{mL}}{150\ \text{mL}} \times 100\% \doteq 17\%$$

c. The maximum possible value is 425 mL, and the minimum possible value is 375 mL.

d. $$\text{Percentage error} = \frac{25\ \text{mL}}{400\ \text{mL}} \times 100\% = 6.25\%$$

Activity 5 (continued)

3. The shape will have to be broken into three rectangles to find the area.



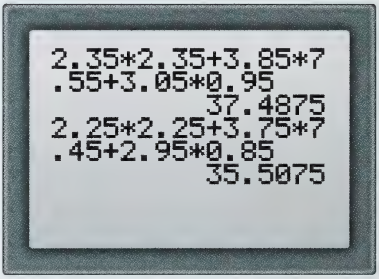
$$\begin{aligned}\text{Area} &= \text{Area } A + \text{Area } B + \text{Area } C\\ &= (2.3\times 2.3)+(3.8\times 7.5)+(0.9\times 3.0)\\ &= 36.5\end{aligned}$$

The uncertainty of each measurement is 0.05 cm.

Calculate the maximum and minimum areas of the shape.

$$\begin{aligned}\text{Maximum area} &= \text{Max Area } A + \text{Max Area } B + \text{Max Area } C\\ &= (2.35\times 2.35)+(3.85\times 7.55)+(3.05\times 0.95)\\ &= 37.4875\ \text{cm}^2\end{aligned}$$

$$\begin{aligned}\text{Minimum area} &= \text{Min Area } A + \text{Min Area } B + \text{Min Area } C\\ &= (2.25\times 2.25)+(3.75\times 7.45)+(2.95\times 0.85)\\ &= 35.5075\ \text{cm}^2\end{aligned}$$



```
2.35*2.35+3.85*7
.55+3.05*0.95
          37.4875
2.25*2.25+3.75*7
.45+2.95*0.85
          35.5075
```

The difference between the maximum and minimum areas is $37.4875-35.5075=1.98\ \text{cm}^2$. Therefore, the uncertainty is $1.98\div 2=0.99\ \text{cm}^2$.

Because the measurements have a precision of 0.1 cm, the area is $(36.5\pm 1.00)\ \text{cm}^2$.

$$\begin{aligned}\therefore\ \text{Percentage error} &= \frac{1.00}{36.5}\times 100\%\\ &= 2.74\%\end{aligned}$$

5. **a.** The diameter of the logo is (3.50 ± 0.05) cm.

b. $$\begin{aligned} A &= \pi r^2 \\ &= \pi\left(\frac{3.50}{2}\right)^2 \\ &\doteq 9.62\ \text{cm}^2 \end{aligned}$$

c. $$\begin{aligned} \text{Percentage error} &= \frac{0.05}{3.5} \times 100\% \\ &\doteq 1.43\% \end{aligned}$$

Since finding the area requires multiplying half of the diameter by itself, the percentage error is doubled. Thus, the percentage error for the area is 2.86%.

Alternately, you could calculate the maximum and minimum areas, and then use that data to find the percentage error.

$$\begin{aligned} \text{Maximum diameter} &= 3.50 + 0.05 \\ &= 3.55\ \text{cm} \end{aligned}$$

$$\begin{aligned} \text{Minimum diameter} &= 3.50 - 0.05 \\ &= 3.45\ \text{cm} \end{aligned}$$

$$\begin{aligned} \text{Maximum area} &= \pi\left(\frac{3.55}{2}\right)^2 \\ &\doteq 9.898\ \text{cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Minimum area} &= \pi\left(\frac{3.45}{2}\right)^2 \\ &\doteq 9.348\ \text{cm}^2 \end{aligned}$$

$$\begin{aligned} \therefore\ \text{Percentage error} &= \frac{\frac{9.898-9.348}{2}}{9.621} \times 100\% \\ &\doteq 2.86\% \end{aligned}$$

2. **Textbook exercises 1 to 10 of "Investigation: The Percentage Error of the Quotient of Two Measurements," p. 388**

Answers will vary, particularly if the experiments were carried out rather than using the given diagrams. The answers given here are for the given diagrams.

1. The mass of the marble is (18.4 ± 0.05) g.

$$\begin{aligned}\therefore \text{ Percentage error (mass, 1 marble)} &= \frac{0.05}{18.4} \times 100\% \\ &\doteq 0.27\%\end{aligned}$$

2. The volume displaced by the marble is (58 ± 0.5) mL $-$ (50 ± 0.5) mL $= (8.0 \pm 1.0)$ mL.

$$\begin{aligned}\therefore \text{ Percentage error (volume, 1 marble)} &= \frac{1}{8} \times 100\% \\ &= 12.5\%\end{aligned}$$

3. Recall that 1 mL of water is equal to 1 cm^3.

$$\begin{aligned}\therefore D &= \frac{M}{V} \\ &= \frac{18.4 \text{ g}}{8.0 \text{ cm}^3} \\ &= 2.3 \text{ g/cm}^3\end{aligned}$$

4. The largest density comes from the largest mass and the smallest volume.

$$\begin{aligned}\therefore D &= \frac{M}{V} \\ &= \frac{18.45 \text{ g}}{7.0 \text{ cm}^3} \\ &\doteq 2.6 \text{ g/cm}^3\end{aligned}$$

5. The smallest density comes from the smallest mass and the largest volume.

$$\begin{aligned}\therefore D &= \frac{M}{V} \\ &= \frac{18.35 \text{ g}}{9.0 \text{ cm}^3} \\ &\doteq 2.0 \text{ g/cm}^3\end{aligned}$$

6. Uncertainty of the density (1 marble) $\doteq \dfrac{2.6-2.0}{2}$

$$\doteq 0.3\ \text{g/cm}^3$$

7. Percentage error (density, 1 marble) $\doteq \dfrac{0.3}{2.3} \times 100\%$

$$\doteq 13\%$$

8. The percentage error of the density is close to the sum of the individual percentage errors of the mass and the volume. $(13\%$ is close to $12.5\% + 0.27\%.)$

9. The mass of the five marbles is (91.0 ± 0.05) g.

The volume of the five marbles is $(91.0 \pm 0.5)\ \text{mL} - (50.0 \pm 0.5)\ \text{mL} = (41.0 \pm 1.0)\ \text{mL}$.

$\therefore$ Percentage error (mass, 5 marbles) $= \dfrac{0.05}{91.0} \times 100\%$

$$\doteq 0.05\%$$

$\therefore$ Percentage error (volume, 5 marbles) $= \dfrac{1.0}{41.0} \times 100\%$

$$\doteq 2.4\%$$

When you divide by an exact number, the percentage error does not change.

$\therefore$ Average mass of a marble $= \dfrac{91.0\ \text{g} \pm 0.05\%}{5}$

$$= 18.2\ \text{g} \pm 0.05\%$$

$\therefore$ Average volume of a marble $= \dfrac{41.0\ \text{g} \pm 2.4\%}{5}$

$$= 8.2\ \text{g} \pm 2.4\%$$

10. The density of the average marble is

$$\begin{aligned} D &= \frac{M}{V} \\ &= \frac{18.2\ \text{g}}{8.2\ \text{cm}^3} \\ &\doteq 2.2\ \text{g/cm}^3 \end{aligned}$$

The largest possible density for the average marble is

$$\begin{aligned} D &= \frac{M}{V} \\ &= \frac{(18.2+0.05\%)\ \text{g}}{(8.2-2.4\%)\ \text{cm}^3} \\ &\doteq \frac{18.2\ \text{g}}{8.0\ \text{cm}^3} \\ &\doteq 2.3\ \text{g/cm}^3 \end{aligned}$$

$$\begin{aligned} 18.2+0.05\% &= 18.2+0.0005\times 18.2 \\ &\doteq 18.2 \end{aligned}$$

$$\begin{aligned} 8.2-2.4\% &= 8.2-0.024\times 8.2 \\ &\doteq 8.0 \end{aligned}$$

The smallest possible density for the average marble is

$$\begin{aligned} D &= \frac{M}{V} \\ &= \frac{(18.2-0.05\%)\ \text{g}}{(8.2+2.4\%)\ \text{cm}^3} \\ &\doteq \frac{18.2\ \text{g}}{8.4\ \text{cm}^3} \\ &\doteq 2.2\ \text{g/cm}^3 \end{aligned}$$

$$\begin{aligned} 18.2-0.05\% &= 18.2-0.0005\times 18.2 \\ &\doteq 18.2 \end{aligned}$$

$$\begin{aligned} 8.2+2.4\% &= 8.2+0.024\times 8.2 \\ &\doteq 8.4 \end{aligned}$$

$$\begin{aligned} \text{Uncertainty of the density} &\doteq \frac{2.3-2.2}{2} \\ &\doteq 0.05\ \text{g/cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Percentage error (density, of average marble)} &\doteq \frac{0.05}{2.2}\times 100\% \\ &\doteq 2.27\% \end{aligned}$$

Again, the sum of the percentage errors for mass and volume is close to the percentage error of the density.

3. a. **Textbook exercises 1 and 5 of "Discussing the Ideas," p. 390**

1. A car travels 109 km at a constant speed in 59.5 min. Here, you would find the speed of the car by dividing the distance by the time. The percentage error is estimated by adding the percentage error of the speed to the percentage error of the time. In this example, each measurement has three significant digits; so, the result should retain three significant digits as well.

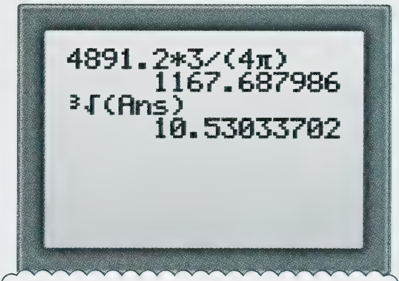
5. You would choose the stack of ten pennies. If you use only one penny, the uncertainty of the measurement would be very close to the height of one penny; therefore, the percentage error would be very large. By using ten pennies, the percentage error would be decreased by a factor of 10.

b. **Textbook exercises 6 and 8 of "Exercises: Checking Your Skills," p. 392**

6. a. Recall that the volume of a sphere is given by the formula, $V = \frac{4}{3}\pi r^3$. Since the maximum volume is $\left(4800 + 0.019 \times 4800\right)\ \text{cm}^3 = 4891.2\ \text{cm}^3$, the maximum radius can be calculated from the volume formula. Similarly, the minimum radius can be calculated from the minimum volume of $\left(4800 - 0.019 \times 4800\right)\ \text{cm}^3 = 4708.8\ \text{cm}^3$.

Find the maxiumum radius.

$$V = \frac{4}{3}\pi r^3$$
$$4891.2 = \left(\frac{4\pi}{3}\right) r^3$$
$$\frac{4891.2 \times 3}{4\pi} = r^3$$
$$r^3 \doteq 1167.687\,986$$
$$r \doteq 10.530$$

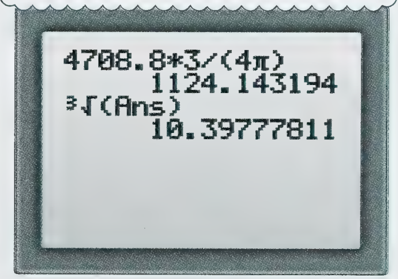


```
4891.2*3/(4π)
         1167.687986
³√(Ans)
         10.53033702
```

To obtain the cube root function on your calculator, press MATH 4.

Find the minimum radius.

$$V = \frac{4}{3}\pi r^3$$
$$4708.8 = \left(\frac{4\pi}{3}\right) r^3$$
$$\frac{4708.8 \times 3}{4\pi} = r^3$$
$$r^3 \doteq 1124.143\,194$$
$$r \doteq 10.398$$



```
4708.8*3/(4π)
         1124.143194
³√(Ans)
         10.39777811
```

Rounded to three decimal places, the maximum radius is 10.530 cm and the minimum radius is 10.398 cm.

b. $r \doteq \dfrac{10.530\text{ cm}+10.398\text{ cm}}{2}$

$\doteq 10.464\text{ cm}$

$\therefore$ Percentage error $\doteq \dfrac{\frac{10.530-10.398}{2}}{10.464} \times 100\%$

$\doteq \dfrac{0.066}{10.464} \times 100\%$

$\doteq 0.631\%$

The radius is about $10.464\text{ cm} \pm 0.631\%$.

c. You would expect the percentage error for the radius to be $\frac{1}{3}$ of the percentage error of the volume. This is true in this example, since $1.9\% \div 3 \doteq 0.63\%$.

8. **a.** Let n be the required number of curb stones needed for one side of the path. The relationship between the number of curb stones, spaces, and the total distance is shown in the equation

$$n(15.60 \pm 0.05)\text{ cm} + (n-1)(1.20 \pm 0.05)\text{ cm} = 270\ 000\text{ cm}$$

The maximum number of curb stones would occur if the spaces and stones were all short (using their minimum lengths).

$$\begin{aligned} \therefore\ n(15.60-0.05)+(n-1)(1.20-0.05) &= 270\ 000 \\ n(15.55)+(n-1)1.15 &= 270\ 000 \\ 15.55n+1.15n-1.15 &= 270\ 000 \\ 16.70n &= 270\ 001.15 \\ n &\doteq 16\ 167.7 \end{aligned}$$

This gives 16 168 whole stones. For both sides of the path, the number is double, or 32 336 stones.

The minimum number of curb stones would occur if the spaces and stones were all long (using their maximum lengths).

$$\begin{aligned} \therefore\ n(15.60+0.05)+(n-1)(1.20+0.05) &= 270\ 000 \\ n(15.65)+(n-1)1.25 &= 270\ 000 \\ 15.65n+1.25n-1.25 &= 270\ 000 \\ 16.90n &= 270\ 001.25 \\ n &\doteq 15\ 976.4 \end{aligned}$$

This gives 15 976 whole stones. For both sides of the path, the number is double, or 31 952 stones.

A good estimate for the number of stones required is the average of the maximum and minimum number.

$$\frac{32\,336+31\,952}{2}=32\,144\ \text{stones}$$

The estimated number of stones required for the walkway is 32 144.

The percentage error is calculated as follows.

$$\begin{aligned}\text{Percentage error} &= \frac{\frac{32\,336-31\,952}{2}}{32\,144}\times 100\% \\ &= \frac{192}{32\,144}\times 100\% \\ &\doteq 0.60\%\end{aligned}$$

b. The maximum cost is calculated using the maximum number of curb stones at the highest price.

$$\begin{aligned}\therefore\ \text{Maximum cost} &= 32\,336\times \$2.10 \\ &= \$67\,905.60\end{aligned}$$

The minimum cost is calculated using the minimum number of curb stones at the lowest price.

$$\begin{aligned}\therefore\ \text{Minimum cost} &= 31\,952\times \$1.70 \\ &= \$54\,318.40\end{aligned}$$

An estimate for the cost is the average of the maximum and minimum costs.

$$\begin{aligned}\text{Estimated cost} &= \frac{\$67\,905.60+\$54\,318.40}{2} \\ &= \$61\,112\end{aligned}$$

$$\begin{aligned}\therefore\ \text{Error in estimated cost} &= \frac{\$67\,905.60-\$54\,318.40}{2} \\ &= \$6793.60\end{aligned}$$

The total estimated cost of lining the walkway is $61 112. The error in the cost estimate is ±$6800.

Note: If you round the estimated number of stones from part a. to 32 000 and multiply by the average price of $1.90, you will get the same estimated cost ($60 800) as is given in the answer in the textbook. Also, the textbook gives the error as a percentage. The error in the answer here is given as a dollar amount.

c. **Textbook exercise 9 of "Exercises: Extending Your Thinking," p. 393**

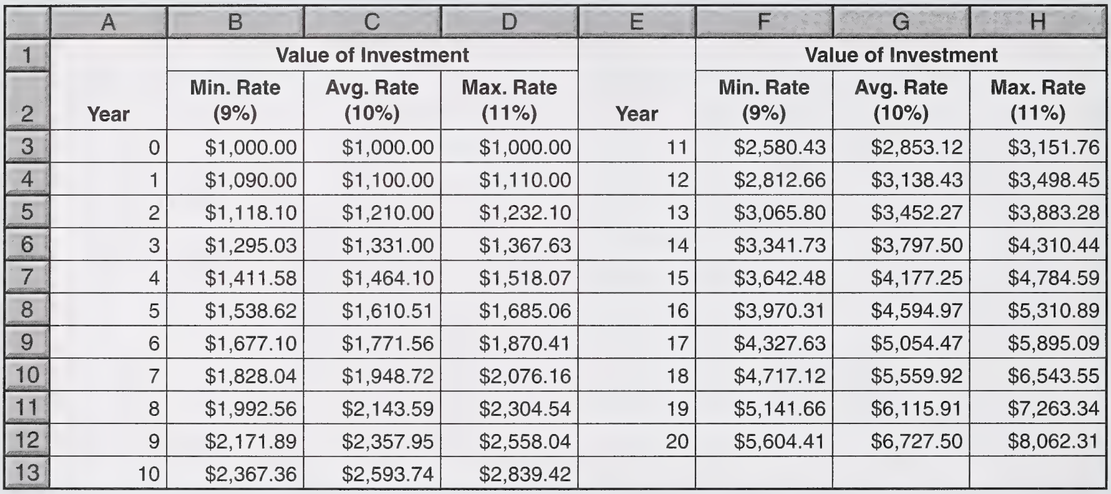
9. The maximum value is calculated by using 11% each of the 20 years.

$$\begin{aligned}\text{Maximum value} &= 1000 \times (1.11)^{20} \\ &\doteq \$8062.31\end{aligned}$$

The minimum value is calulated by using 9% for each of the 20 years.

$$\begin{aligned}\text{Minimum value} &= 1000 \times (1.09)^{20} \\ &\doteq \$5604.41\end{aligned}$$

You could use a spreadsheet to find the values.



	A	B	C	D	E	F	G	H
1		Value of Investment				Value of Investment		
2	Year	Min. Rate (9%)	Avg. Rate (10%)	Max. Rate (11%)	Year	Min. Rate (9%)	Avg. Rate (10%)	Max. Rate (11%)
3	0	$1,000.00	$1,000.00	$1,000.00	11	$2,580.43	$2,853.12	$3,151.76
4	1	$1,090.00	$1,100.00	$1,110.00	12	$2,812.66	$3,138.43	$3,498.45
5	2	$1,118.10	$1,210.00	$1,232.10	13	$3,065.80	$3,452.27	$3,883.28
6	3	$1,295.03	$1,331.00	$1,367.63	14	$3,341.73	$3,797.50	$4,310.44
7	4	$1,411.58	$1,464.10	$1,518.07	15	$3,642.48	$4,177.25	$4,784.59
8	5	$1,538.62	$1,610.51	$1,685.06	16	$3,970.31	$4,594.97	$5,310.89
9	6	$1,677.10	$1,771.56	$1,870.41	17	$4,327.63	$5,054.47	$5,895.09
10	7	$1,828.04	$1,948.72	$2,076.16	18	$4,717.12	$5,559.92	$6,543.55
11	8	$1,992.56	$2,143.59	$2,304.54	19	$5,141.66	$6,115.91	$7,263.34
12	9	$2,171.89	$2,357.95	$2,558.04	20	$5,604.41	$6,727.50	$8,062.31
13	10	$2,367.36	$2,593.74	$2,839.42				

$$\begin{aligned}\therefore\ \text{Estimated value} &= \frac{\$8062.31 + \$5604.41}{2} \\ &= \$6833.36\end{aligned}$$

$$\begin{aligned}\therefore\ \text{Estimated error} &= \frac{\$8062.31 - \$5604.41}{2} \\ &= \$1228.95\end{aligned}$$

The estimated total value after 20 years is $\$6833.36 \pm \$1228.95$.

4. **Textbook exercises 3 and 4 of "Discussing the Ideas," p. 390**

   3. Because the area is given with three significant digits, the calculated radius should also display three significant digits.

   4. The value for π stored in your calculator will give the most precise value that can be used with that calculator. It should be used to minimize errors introduced by an imprecise value of π.

5. **a.** There are 3 significant digits.

   b. There is only 1 significant digit according to the example in the textbook (page 389).

   c. There are 3 significant digits.

   d. There are 2 significant digits.

   e. There are 4 significant digits.

   f. There are 3 significant digits.

   g. There are 3 significant digits.

6. **Textbook exercise "Communicating the Ideas," p 393**

   Answers will vary. A sample answer of the steps involved for estimating the volume of a cylinder is given.

   Step 1: Measure the diameter of the cylinder using a metric ruler with a precision of 0.1 cm.

   Step 2: Calculate the percentage error of the diameter by performing the calculation $\frac{0.05}{d} \times 100\%$. The percentage error for the radius will be the same value.

   Step 3: Measure the height of the cylinder using the same ruler.

   Step 4: Calculate the percentage error of the height by performing the calculation $\frac{0.05}{h} \times 100\%$.

   Step 5. Use the formula $V = \pi r^2 h$ to find the nominal value for the volume of the cylinder.

   Step 6: Estimate the percentage error of the volume by adding the percentage error of the height to twice the percentage error of the radius. Because the radius is multiplied by itself in calculating the volume, its error needs to be included two times.

   Step 7: Write the volume using ± notation, making sure to include only the proper number of significant digits and the error estimate from Step 6.

Activity 6: How Close Is Close Enough?

1. The specifications give the acceptable lengths as (12.0 ± 0.1) cm. Therefore, the maximum allowable length is $12.0 \text{ cm} + 0.1 \text{ cm} = 12.1 \text{ cm}$. Any part longer than this is too long to be acceptable.

Likewise, the minimum allowable length is $12.0 \text{ cm} - 0.1 \text{ cm} = 11.9 \text{ cm}$. Any part shorter than this is too short to be acceptable.

2. a. Textbook exercises 1 to 4 of "Discussing the Ideas," p. 397

1. Both tolerances and uncertainties use the ± notation. Tolerances are allowed variations in the size of objects related to the specifications for the building of the object. Uncertainty relates to the precision of the measurements made on objects.

2. The maximum measurement is obtained by adding the tolerance. The minimum measurement is obtained by subtracting the tolerance.

3. In Example 1, exceeding the tolerances would probably make the bulletproof vest less than bulletproof, possibly leading to the death of the person wearing the vest.

In Example 2, exceeding tolerances would make the door unsafe for people who would be using the elevator.

In Example 3, exceeding tolerances could lead to disqualification or, perhaps, allow a piston to hit a valve and destroy the engine.

4. Tolerances are used in all areas of life to help ensure safety and prevent malfunction of the products produced.

b. Textbook exercises 1 to 4 of "Exercises: Checking Your Skills," p. 398

1. a. The maximum acceptable measurement is $3.75 + 0.05 = 3.80$ mm.

The minimum acceptable measurement is $3.75 - 0.05 = 3.70$ mm.

b. The maximum acceptable measurement is $0.0080 + 0.0002 = 0.0082$ inches.

The minimum acceptable measurement is $0.0080 - 0.0002 = 0.0078$ inches.

2. **a.** $\text{Tolerance range} = 8.5 - 7.9$

$$= 0.6 \text{ g}$$

$\text{Tolerance} = 0.6 \div 2$

$$= \pm 0.3 \text{ g}$$

$$\text{Centre of tolerance range} = \frac{7.9 + 8.5}{2}$$

$$= 8.2 \text{ g}$$

Therefore, the tolerance interval is (8.2 ± 0.3) g.

b. $\text{Tolerance range} = 3.1 - 3.0$

$$= 0.1 \text{ cubic feet}$$

$\text{Tolerance} = 0.1 \div 2$

$$= 0.05 \text{ cubic feet}$$

$$\text{Centre of tolerance range} = \frac{3.1 + 3.0}{2}$$

$$= 3.05 \text{ cubic feet}$$

Therefore, the tolerance interval is (3.05 ± 0.05) cubic feet.

3. $\frac{55}{64}$ inches $= 0.859\ 375$ inches

This value falls within the tolerances specified (0.8585 inches to 0.8595 inches).

4. The tolerance interval for the metal rod is 2.8 cm to 3.2 cm.

a. 3.3 cm is outside the tolerance interval.
b. 2.08 cm is outside the tolerance interval.
c. 3.09 cm is within the tolerance interval.
d. 2.98 cm is within the tolerance interval.

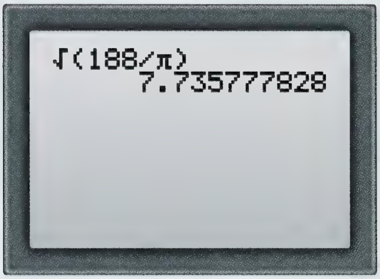
Activity 6 (continued)

c. Textbook exercises 7 and 8 of “Exercises: Extending Your Thinking,” p. 399

7. The acceptable cross-sectional areas are between $43\ \text{mm}^2$ and $47\ \text{mm}^2$. To find the acceptable diameters, you will need to use the formula $A = \pi\left(\frac{d}{2}\right)^2$, which relates the area to the diameter of a circular object.

Find the maximum diameter.

$$\begin{aligned} A_{\max} &= \pi\left(\frac{d}{2}\right)^2 \\ 47 &= \frac{\pi d^2}{4} \\ 188 &= \pi d^2 \\ d &= \sqrt{\frac{188}{\pi}} \\ &\doteq 7.7 \end{aligned}$$

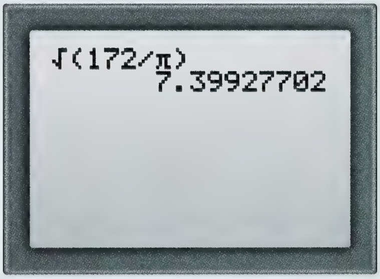


```
√(188/π)
         7.735777828
```

The maximum acceptable diameter of the wire is 7.7 mm.

Find the minimum diameter.

$$\begin{aligned} A_{\min} &= \pi\left(\frac{d}{2}\right)^2 \\ 43 &= \frac{\pi d^2}{4} \\ 172 &= \pi d^2 \\ d &= \sqrt{\frac{172}{\pi}} \\ &\doteq 7.4 \end{aligned}$$



```
√(172/π)
          7.39927702
```

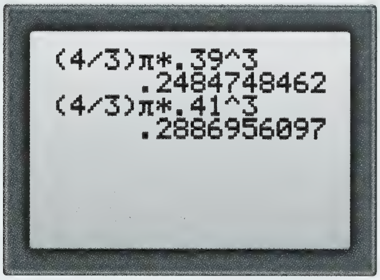
The minimum acceptable diameter of the wire is 7.4 mm.

8. The volume of a sphere based on the diameter is found using the formula $V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$. The maximum diameter is 0.82 cm, and the minimum diameter is 0.78 cm.

Calculate the minimum and maximum volumes.

$$\begin{aligned} V_{\text{min}} &= \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \\ &= \frac{4}{3}\pi\left(\frac{0.78}{2}\right)^3 \\ &= \frac{4}{3}\pi(0.39)^3 \\ &\doteq 0.25 \end{aligned}$$

$$\begin{aligned} V_{\text{max}} &= \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \\ &= \frac{4}{3}\pi\left(\frac{0.82}{2}\right)^3 \\ &= \frac{4}{3}\pi(0.41)^3 \\ &\doteq 0.29 \end{aligned}$$



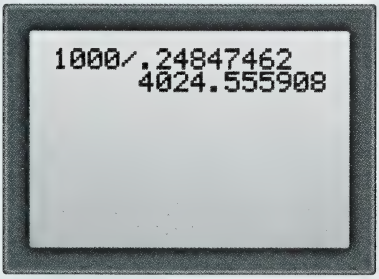
```
(4/3)π*.39^3
          .2484748462
(4/3)π*.41^3
          .2886956097
```

The tolerance range is $0.29 - 0.25 = 0.04$. The tolerance is half of this value, thus making it ± 0.02.

Because there are only two significant digits in the diameter measurements, the reported volume and tolerance measurement should be rounded to the same number of significant digits.

The nominal volume is $0.27\ \text{cm}^3$. This gives a volume of $(0.27 \pm 0.02)\ \text{cm}^3$.

The assumption is that the maximum number of ball bearings that can be made from $1000\ \text{cm}^3$ of steel would be calculated from the smallest volume of one ball bearing. This leads to the following calculation.



```
1000/.24847462
          4024.555908
```

The number of ball bearings that can be made from $1000\ \text{cm}^3$ of steel is 4024.

Note: You may have used a rounded volume of $0.25\ \text{cm}^3$, which would give a maximum number of 4000 ball bearings. You should, however, use unrounded values until you reach your final answer. It is at that point that rounding should take place.

Activity 6 (continued)

3. Textbook exercise "Communicating the Ideas," p. 399

Answers will vary. A sample answer is given.

Tolerances play a major role in most of the human body's systems. Consider blood pressure, blood sugar (hyperglycemia or hypoglycemia), eye ball pressures (glaucoma), and even body temperature (hyperthermia or hypothermia). If any of these fall outside a fairly narrow range, serious medical problems are likely to occur. You may wish to research these conditions. There are a number of reputable Internet sites that discuss them.

Follow-up Activities

Textbook exercises 2, 3.a., 4, 5, 6, 8, 9, and 10 of Part B of "What Should I Be Able to Do?," pp. 404 and 405

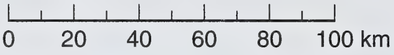
2. a. ratio form

$$1 \text{ km} = 100\,000 \text{ cm}$$

$$\therefore \frac{1 \text{ cm}}{20 \text{ km}} = \frac{1 \text{ cm}}{20 \text{ km} \times 100\,000 \text{ cm/km}}$$

$$= \frac{1}{2\,000\,000}$$

linear form

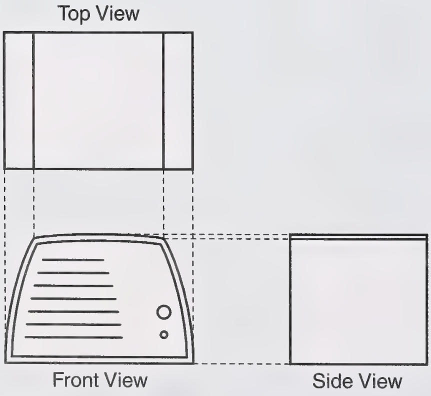


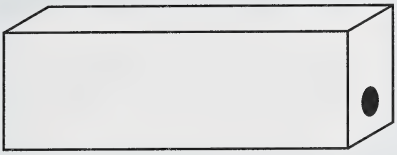
b. Nanaimo to Port Alberni: $3.3 \text{ cm} \times \frac{20 \text{ km}}{1 \text{ cm}} = 66 \text{ km}$

Prince George to Kelowna: $26.3 \text{ cm} \times \frac{20 \text{ km}}{1 \text{ cm}} = 526 \text{ km}$

Dawson Creek to Prince Rupert: $34.7 \text{ cm} \times \frac{20 \text{ km}}{1 \text{ cm}} = 694 \text{ km}$

3. a.



4. 

5. a. The uncertainty for 37.23 mm is 0.005 mm.

The uncertainty for 37.4 mm is 0.05 mm.

The uncertainty for 3.7 cm is 0.05 cm.

The uncertainty for 4 cm is 0.5 cm.

b. $\frac{0.005\ \text{mm}}{37.23\ \text{mm}} \times 100\% \doteq 0.0134\%$

$\therefore\ 37.23\ \text{mm} \pm 0.0134\%$

$\frac{0.05\ \text{mm}}{37.4\ \text{mm}} \times 100\% \doteq 0.134\%$

$\therefore\ 37.4\ \text{mm} \pm 0.134\%$

$\frac{0.05\ \text{cm}}{3.7\ \text{cm}} \times 100\% \doteq 1.35\%$

$\therefore\ 3.7\ \text{cm} \pm 1.35\%$

$\frac{0.5\ \text{cm}}{4\ \text{cm}} \times 100\% = 12.5\%$

$\therefore\ 4\ \text{cm} \pm 12.5\%$

6. a. The stem measures 1.4 cm.

b. The uncertainty is 0.05 cm. The measurement is (1.4 ± 0.05) cm.

c. If 0.8 cm exactly were removed from this length, the maximum possible remaining length would be 0.65 cm. The minimum possible remaining length would be 0.55 cm.

8. $(32.4 - 31.8)\ \text{mg} = 0.6\ \text{mg}$

Split this tolerance range in half to obtain the tolerance of $\pm 0.3\ \text{mg}$. Use the average of the boundary values as the nominal value.

$$\frac{32.4 + 31.8}{2} = 32.1\ \text{mg}$$

The tolerance interval is (32.1 ± 0.3) mg.

9. The maximum acceptable difference comes from the largest value for A and the smallest value for B. This difference is $8.24\ \text{mm} - 4.24\ \text{mm} = 4.00\ \text{mm}$.

The minimum acceptable difference comes from the smallest value for A and the largest value for B. This difference is $8.16\ \text{mm} - 4.36\ \text{mm} = 3.80\ \text{mm}$.

Follow-up Activities (continued)

10. The maximum distance is 7.5 km; the minimum distance is 6.5 km; the maximum speed is 4.25 km/h; and the minimum speed is 3.75 km/h.

The smallest possible time is calculated using the minimum distance and the greatest speed.

$$\frac{6.5\ \text{km}}{4.25\ \text{km/h}} \doteq 1.529\ \text{h}$$

Since one measurement in the question has two significant digits and the other three significant digits, the result should be reported as 1.5 h.

The largest possible time is calculated using the maximum distance and the slowest speed.

$$\frac{7.5\ \text{km}}{3.75\ \text{km/h}} = 2\ \text{h}$$

Since one measurement in the question has two significant digits and the other three significant digits, the result should be reported as 2.0 h.

Extra Help

1. The maximum width is 30.05 cm, and the maximum length is 17.05 cm.

$$\begin{aligned}\therefore A_{\text{max}} &= 30.05 \times 17.05\\ &= 512.3525\ \text{cm}^2\end{aligned}$$

The minimum width is 29.95 cm, and the minimum length is 16.95 cm.

$$\begin{aligned}\therefore A_{\text{min}} &= 29.95 \times 16.95\\ &= 507.6525\ \text{cm}^2\end{aligned}$$

$$\begin{aligned}\text{Nominal area} &= \frac{512.3525 + 507.6525}{2}\\ &= 510.0025\ \text{cm}^2\end{aligned}$$

The amount of error is $512.3525 - 507.6525 = 4.7000\ \text{cm}^2$. Using ± notation, the error is $\pm 2.3500\ \text{cm}^2$.

Using the same precision as was used when the measurements were taken, the area is $(510.00 \pm 2.35)\ \text{cm}^2$.

The precentage is calculated as follows.

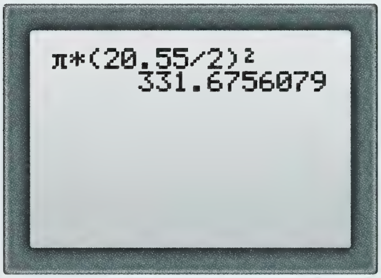
$$\text{Percentage error} = \frac{2.3500}{510.0025} \times 100\% \\ \doteq 0.46\%$$

$$\text{Percentage error} = \frac{\text{Tolerance}}{\text{Nominal area}} \times 100\%$$

Using the same precision as was used when the measurements were taken, the area is $510.00 \text{ cm}^2 \pm 0.46\%$.

2. The area of a circle can be calculated from the diameter using the formula $A = \pi\left(\frac{d}{2}\right)^2$. The maximum possible area is calculated using the maximum possible diameter.

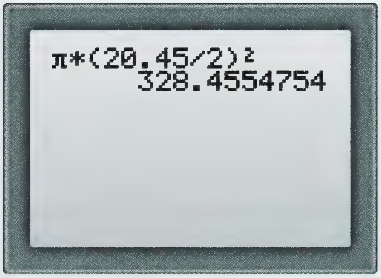
$$\begin{aligned} \therefore A_{\text{max}} &= \pi\left(\frac{d}{2}\right)^2 \\ &= \pi\left(\frac{20.55}{2}\right)^2 \\ &\doteq 331.676 \text{ cm}^2 \end{aligned}$$



```
π*(20.55/2)²
          331.6756079
```

The minimum possible area is calculated using the minimum possible diameter.

$$\begin{aligned} \therefore A_{\text{min}} &= \pi\left(\frac{d}{2}\right)^2 \\ &= \pi\left(\frac{20.45}{2}\right)^2 \\ &\doteq 328.455 \text{ cm}^2 \end{aligned}$$



```
π*(20.45/2)²
          328.4554754
```

$$\begin{aligned} \text{Nominal area} &\doteq \frac{331.676 + 328.455}{2} \\ &\doteq 330.066 \text{ cm}^2 \end{aligned}$$

The amount of error is $331.676 - 328.455 \doteq 3.221 \text{ cm}^2$. Using ± notation, the error is $\pm 1.611 \text{ cm}^2$. Using the same precision as was used when the measurements were taken, the area is $(330.07 \pm 1.61) \text{ cm}^2$.

The percentage error is calculated as follows:

$$\text{Percentage error} \doteq \frac{1.611}{330.066} \times 100\% \\ \doteq 0.49\%$$

$$\text{Percentage error} = \frac{\text{Tolerance}}{\text{Nominal area}} \times 100\%$$

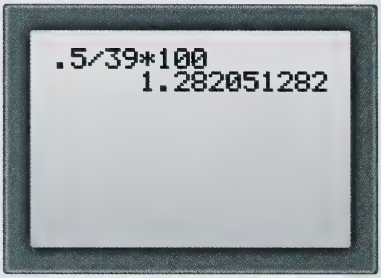
Using the same precision as was used when the measurements were taken, the area is $330.07 \text{ cm}^2 \pm 0.49\%$.

Follow-up Activities (continued)

Enrichment

Because the precision of the measurement is 1 AU, the percentage error is calculated from half of the precision of the measurement (assuming the error is half of the precision).

$$\therefore \text{ Percentage error} = \frac{0.5 \text{ AU}}{39 \text{ AU}} \times 100\%$$
$$\doteq 1.28\%$$



```
.5/39*100
         1.282051282
```

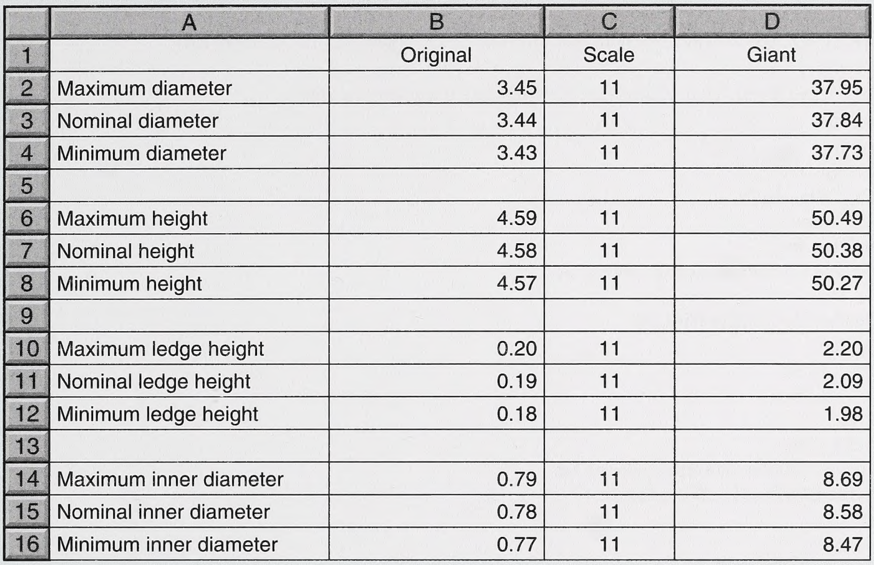
The distance from Pluto to the Sun, is $39 \text{ AU} \pm 1.3\%$.

Module Project: Building a Giant Replica

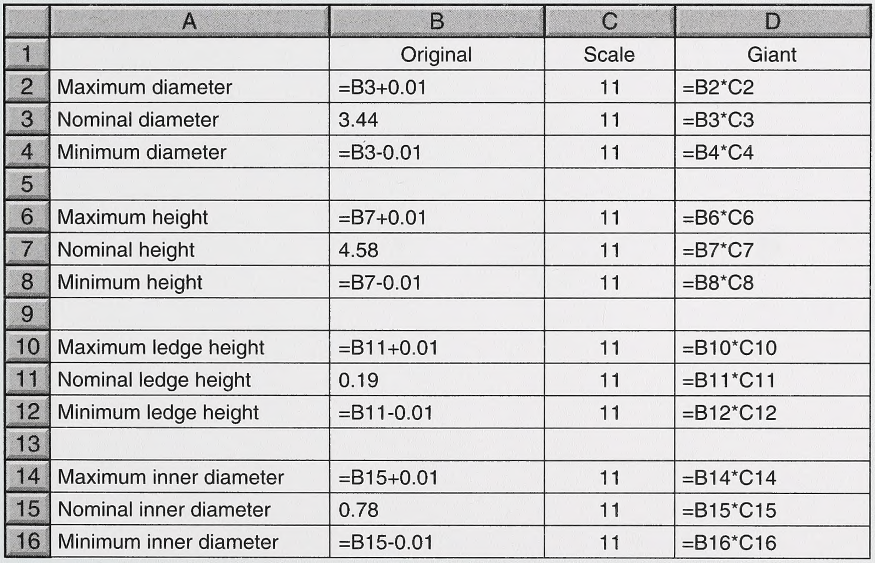
Textbook exercises 11 and 12 of Part C of "What Should I Be Able to Do?," pp. 407 and 408

11. The scale is shown as 200%, but no original is shown. So, it is not possible to judge if this drawing is correct. Measurements for most of the significant features are shown. The outer diameter of the centre part of the spool is not shown; so, the thickness of this wall is not known. Also, the top lip of the spool has no dimensions; it may be the same as the bottom lip. There are no dashed lines showing how the drawings align, and there are no dashed lines showing the internal parts in the top view.

12. The scale is not stated explicitly, but it appears to be 11:1 (as shown in the lower spreadsheet on page 408 of the textbook). The calculations are probably correct since they were done by a spreadsheet program (although the formulas used in the calculations could be wrong). These are very confusing spreadsheets. A much better layout of the data and calculations is as follows.



	A	B	C	D
1		Original	Scale	Giant
2	Maximum diameter	3.45	11	37.95
3	Nominal diameter	3.44	11	37.84
4	Minimum diameter	3.43	11	37.73
5				
6	Maximum height	4.59	11	50.49
7	Nominal height	4.58	11	50.38
8	Minimum height	4.57	11	50.27
9				
10	Maximum ledge height	0.20	11	2.20
11	Nominal ledge height	0.19	11	2.09
12	Minimum ledge height	0.18	11	1.98
13				
14	Maximum inner diameter	0.79	11	8.69
15	Nominal inner diameter	0.78	11	8.58
16	Minimum inner diameter	0.77	11	8.47



	A	B	C	D
1		Original	Scale	Giant
2	Maximum diameter	=B3+0.01	11	=B2*C2
3	Nominal diameter	3.44	11	=B3*C3
4	Minimum diameter	=B3-0.01	11	=B4*C4
5				
6	Maximum height	=B7+0.01	11	=B6*C6
7	Nominal height	4.58	11	=B7*C7
8	Minimum height	=B7-0.01	11	=B8*C8
9				
10	Maximum ledge height	=B11+0.01	11	=B10*C10
11	Nominal ledge height	0.19	11	=B11*C11
12	Minimum ledge height	=B11-0.01	11	=B12*C12
13				
14	Maximum inner diameter	=B15+0.01	11	=B14*C14
15	Nominal inner diameter	0.78	11	=B15*C15
16	Minimum inner diameter	=B15-0.01	11	=B16*C16

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